# Part II:
# Structure Formation

# Non-relativistic Ideal Fluid

(baryonic gas, collision-less DM)

$$\boxed{\begin{aligned} &\frac{\mathrm{D}\rho}{\mathrm{D}t} + \rho\nabla_{\mathbf{r}}\cdot\mathbf{u} = 0 \quad \text{(continuity)},\\ &\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} = -\frac{\nabla_{\mathbf{r}}P}{\rho} - \nabla_{\mathbf{r}}\phi \quad \text{(Euler)},\\ &\nabla^2_{\mathbf{r}}\phi = 4\pi G\rho \quad \text{(Poisson)}, \end{aligned}}$$

(◎)

$\mathbf{r}$ : proper coordinate

$$\boxed{\mathbf{r} = a(t)\mathbf{x}.}$$

$\mathbf{x}$ : comoving coord.

5 eqns, 6 unknowns:

$(\rho, u_x, u_y, u_z, P, \text{and } \phi)$.

$\longrightarrow$ add **EOS**

Lagrangian, convective derivative: $\frac{\mathrm{D}}{\mathrm{D}t} \equiv \frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla_{\mathbf{r}}$

Proper velocity:
$$\begin{aligned} \mathbf{u} &= \dot{\mathbf{r}} \\ &= \dot{a}\mathbf{x} + a\dot{\mathbf{x}} \\ &= H\mathbf{r} + a\dot{\mathbf{x}} \\ &= \mathbf{v}_{\mathrm{H}} + \mathbf{v}_{\mathrm{p}} \end{aligned}$$

$$\mathbf{v}_{\mathrm{H}} = H\,\mathbf{r} = \left(\frac{\dot{a}}{a}\right)\mathbf{r}$$

**Hubble flow**

$$\mathbf{v}_{\mathrm{p}} \equiv a\dot{\mathbf{x}},$$

**Peculiar velocity**

# Extension to include other background fluids:

$$\nabla^2_{\mathbf{r}}\phi = 4\pi G\rho \quad \text{(Poisson)},$$

$$\nabla^2_{\mathbf{r}}\phi = 4\pi G\left(\rho + \tilde{\rho}_{\rm r} + \tilde{\rho}_{\rm v}\right),$$

equiv. mass density for

$\tilde{\rho}_{\rm r}$ : relativistic background

$\tilde{\rho}_{\rm v}$ : vacuum energy

general source term

$$\tilde{\rho} = \rho + 3P/c^2.$$

$$\begin{cases} \tilde{\rho}_{\rm r} = \rho_{\rm r} + 3P_{\rm r}/c^2 = 2\rho_{\rm r}, \\ \tilde{\rho}_{\rm v} = \rho_{\rm v} + 3P_{\rm v}/c^2 = -2\rho_{\rm v}. \end{cases}$$

Remember, $\dfrac{\ddot{a}}{a} = -\dfrac{4\pi G}{3}\left(\rho + 3\dfrac{P}{c^2}\right) + \dfrac{\Lambda c^2}{3}$

So, the contrib. of background can be subtracted off as: $\phi = \Phi - a\ddot{a}x^2/2,$

**Effect of adding background is to change only a(t).**

# Linear Perturbation Theory

From (◎), coordinate change:

$$(\mathbf{r}, t) \longrightarrow (\mathbf{x}, t)$$

$$\nabla_{\mathbf{r}} \rightarrow \frac{1}{a}\nabla_{\mathbf{x}}\,; \qquad \frac{\partial}{\partial t} \rightarrow \frac{\partial}{\partial t} - \frac{\dot{a}}{a}\mathbf{x}\cdot\nabla_{\mathbf{x}}.$$

Density: $\rho(\mathbf{x},t) = \overline{\rho}(t)\,[1+\delta(\mathbf{x},t)]\,,$ Overdensity: $\delta = \dfrac{\rho - \bar{\rho}}{\bar{\rho}}$

$$\boxed{\begin{aligned} &\frac{\partial\delta}{\partial t} + \frac{1}{a}\nabla\cdot[(1+\delta)\mathbf{v}] = 0,\\ &\frac{\partial\mathbf{v}}{\partial t} + \frac{\dot{a}}{a}\mathbf{v} + \frac{1}{a}(\mathbf{v}\cdot\nabla)\mathbf{v} = -\frac{\nabla\Phi}{a} - \frac{\nabla P}{a\overline{\rho}(1+\delta)}, \quad (☆)\\ &\nabla^2\Phi = 4\pi G\overline{\rho}a^2\delta, \quad \Phi \equiv \phi + a\ddot{a}x^2/2, \end{aligned}}$$

$$\nabla \equiv \nabla_{\mathbf{x}},$$

plus

Eq. of. State: $P = P(\rho, S),$

Jeans, Lifshitz, Peebles, Mo, …

Specific entropy: $\mathrm{d}S = \mathrm{d}Q/T$

1st law of thermodynamics for unit mass: $T\,\mathrm{d}S = \mathrm{d}\left(\frac{3}{2}\frac{P}{\rho}\right) + P\,\mathrm{d}\left(\frac{1}{\rho}\right).$

$$P = (\rho/\mu m_\mathrm{p})k_\mathrm{B}T,$$

$$\mathrm{d}\ln P = \frac{5}{3}\mathrm{d}\ln\rho + \frac{2}{3}\frac{\mu m_\mathrm{p}}{k_\mathrm{B}}S\,\mathrm{d}\ln S, \longrightarrow P \propto \rho^{5/3}\exp\left(\frac{2}{3}\frac{\mu m_\mathrm{p}}{k_\mathrm{B}}S\right).$$

Thus,
$$\frac{\nabla P}{\bar{\rho}} = \frac{1}{\bar{\rho}}\left[\left(\frac{\partial P}{\partial \rho}\right)_S \nabla\rho + \left(\frac{\partial P}{\partial S}\right)_\rho \nabla S\right]$$
$$= c_\mathrm{s}^2\nabla\delta + \frac{2}{3}(1+\delta)T\nabla S,$$

adiabatic sound speed:

$$c_\mathrm{s} = \left(\frac{\partial P}{\partial \rho}\right)_S^{1/2}$$

**Euler eq. (☆) becomes:**

$$\frac{\partial \mathbf{v}}{\partial t} + \frac{\dot{a}}{a}\mathbf{v} + \frac{1}{a}(\mathbf{v}\cdot\nabla)\mathbf{v} = -\frac{\nabla\Phi}{a} - \frac{c_\mathrm{s}^2}{a}\frac{\nabla\delta}{(1+\delta)} - \frac{2T}{3a}\nabla S.$$

Neglecting nonlinear terms:

$$\boxed{\begin{aligned}&\frac{\partial\delta}{\partial t} + \frac{1}{a}\nabla\cdot\mathbf{v} = 0, \quad (\bigstar)\\ &\frac{\partial \mathbf{v}}{\partial t} + \frac{\dot{a}}{a}\mathbf{v} = -\frac{\nabla\Phi}{a} - \frac{c_\mathrm{s}^2}{a}\nabla\delta - \frac{2\overline{T}}{3a}\nabla S,\end{aligned}}$$

Differentiate (★) and using earlier eqns:

$$\boxed{\frac{\partial^2 \delta}{\partial t^2} + 2\frac{\dot{a}}{a}\frac{\partial \delta}{\partial t} = 4\pi G\bar{\rho}\delta + \frac{c_s^2}{a^2}\nabla^2\delta + \frac{2}{3}\frac{\overline{T}}{a^2}\nabla^2 S.}$$

Hubble drag

gravity

pressures

Fourier transform:

$$\delta(\mathbf{x},t) = \sum_{\mathbf{k}} \delta_{\mathbf{k}}(t)\exp(i\mathbf{k}\cdot\mathbf{x})\,; \qquad \delta_{\mathbf{k}}(t) = \frac{1}{V_u}\int \delta(\mathbf{x},t)\exp(-i\mathbf{k}\cdot\mathbf{x})\,\mathrm{d}^3\mathbf{x},$$

**k**: comoving wave vector

$$\frac{\mathrm{d}^2\delta_{\mathbf{k}}}{\mathrm{d}t^2} + 2\frac{\dot{a}}{a}\frac{\mathrm{d}\delta_{\mathbf{k}}}{\mathrm{d}t} = \left[4\pi G\bar{\rho} - \frac{k^2 c_s^2}{a^2}\right]\delta_{\mathbf{k}} - \frac{2}{3}\frac{\overline{T}}{a^2}k^2 S_{\mathbf{k}}.$$

$$-k^2\Phi_{\mathbf{k}} = 4\pi G\bar{\rho}a^2\delta_{\mathbf{k}},$$

$$\mathbf{v} = \nabla\mathcal{V}, \quad \mathbf{v}_{\mathbf{k}} = i\mathbf{k}\mathcal{V}_{\mathbf{k}}. \qquad \mathbf{v}_{\mathbf{k}} = \frac{ia\mathbf{k}}{k^2}\frac{\mathrm{d}\delta_{\mathbf{k}}}{\mathrm{d}t}.$$

velocity potential

## For `isentropic' initial perturbation, $k^2 S_{\mathbf{k}} = 0.$

$$\boxed{\frac{d^2\delta_{\mathbf{k}}}{dt^2} + 2H(t)\frac{d\delta_{\mathbf{k}}}{dt} = \delta_{\mathbf{k}}\left(4\pi G\bar{\rho}(t) - \frac{k^2 c_s^2}{a(t)^2}\right).}$$

gravity ⟷ pressure

**Jeans wavelength** $\lambda_J = \frac{2\pi a(t)}{k_J} = c_s\left(\frac{\pi}{G\bar{\rho}}\right)^{1/2}.$

$\boxed{M > M_J \quad \text{or} \quad \lambda > \lambda_J \quad \text{or} \quad k < k_J}$ then, **collapse**.

**Jeans mass** $\boxed{M_J = \frac{4\pi}{3}\rho\left(\frac{\lambda_J}{2}\right)^3.}$

$$M_J = \frac{4\pi}{3}\rho_m\left(\frac{\pi}{k_J}\right)^3 = 4\times 10^9\left(\frac{T}{10^4\ \mathrm{K}}\right)^{3/2}\left(\frac{n_H}{10^{-3}\ \mathrm{cm}^{-3}}\right)^{-1/2} \mathrm{M}_\odot$$

where $k_J \equiv c_s^{-1} t_{\mathrm{dyn}}^{-1}, \qquad t_{\mathrm{dyn}} \equiv (4\pi G\rho_m)^{-1/2}$

$\rho_m$ : total mass density

## Specific solutions:

(a) Pressure-less fluid:

$$\boxed{\frac{d^2\delta_{\mathbf{k}}}{dt^2} + 2\frac{\dot{a}}{a}\frac{d\delta_{\mathbf{k}}}{dt} = 4\pi G\bar{\rho}_m \delta_{\mathbf{k}},}$$

- Einstein-de-Sitter Univ.: $a(t) \propto t^{2/3}$.

$\Omega_m = 1 \qquad \Lambda = 0$

$$\delta_+ \propto t^{2/3}; \quad \delta_- \propto H(t) \propto t^{-1},$$

- Open Univ.:

$(\Omega_{m,0} < 1,\ \Omega_{\Lambda,0} = 0)$

$k = -1$

$$\delta_+ \propto 1 + \frac{3}{x} + \frac{3(1+x)^{1/2}}{x^{3/2}} \ln\left[(1+x)^{1/2} - x^{1/2}\right],$$

where $x \equiv (\Omega_{m,0}^{-1} - 1)/(1+z)$.

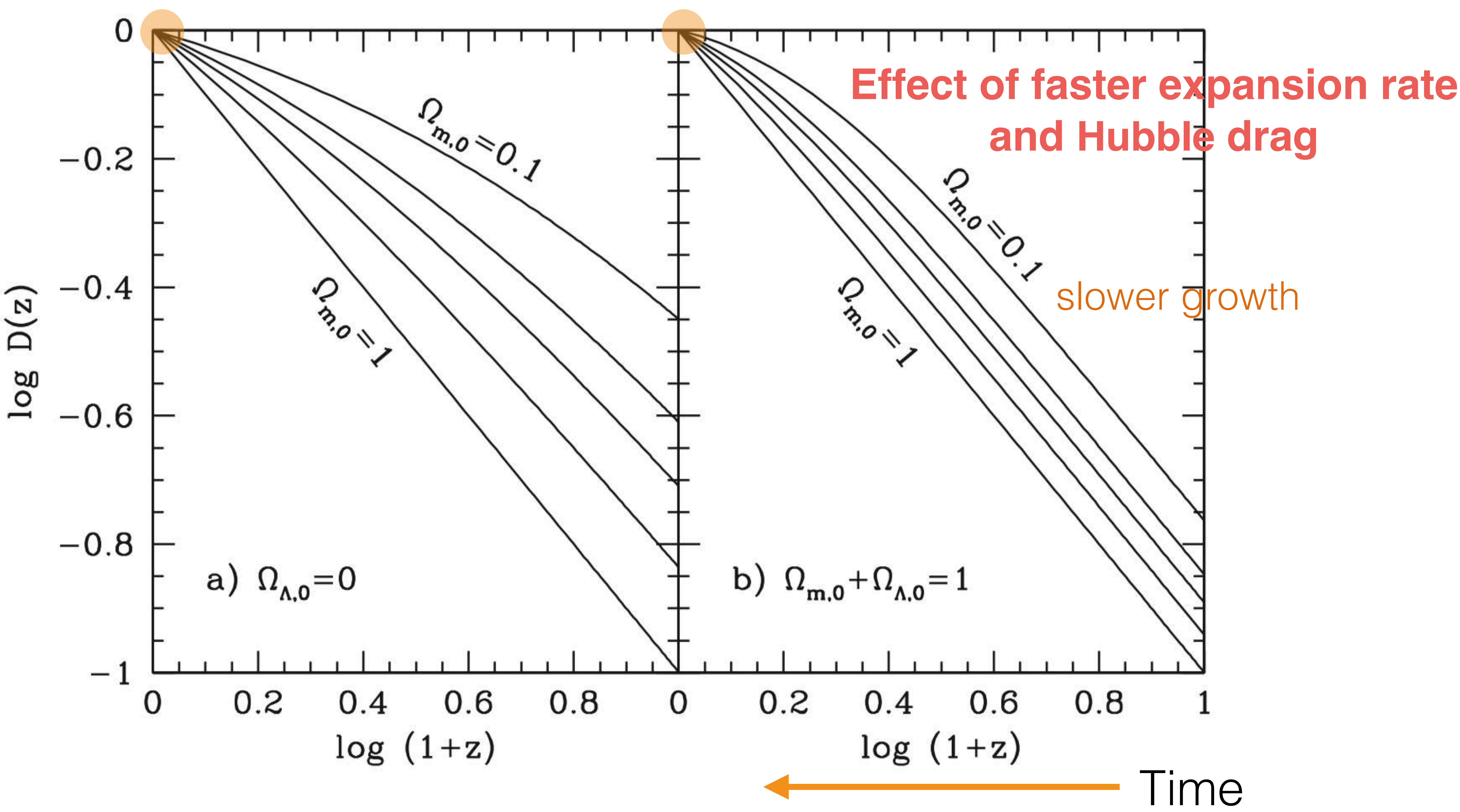
$\delta_+ \propto (1+z)^{-1}$ as $x \to 0$ $\qquad$ $\delta_+ \to 1$ as $x \to \infty$.

- In general, $\delta_+ \propto D(z) \propto g(z)/(1+z)$, linear growth rate

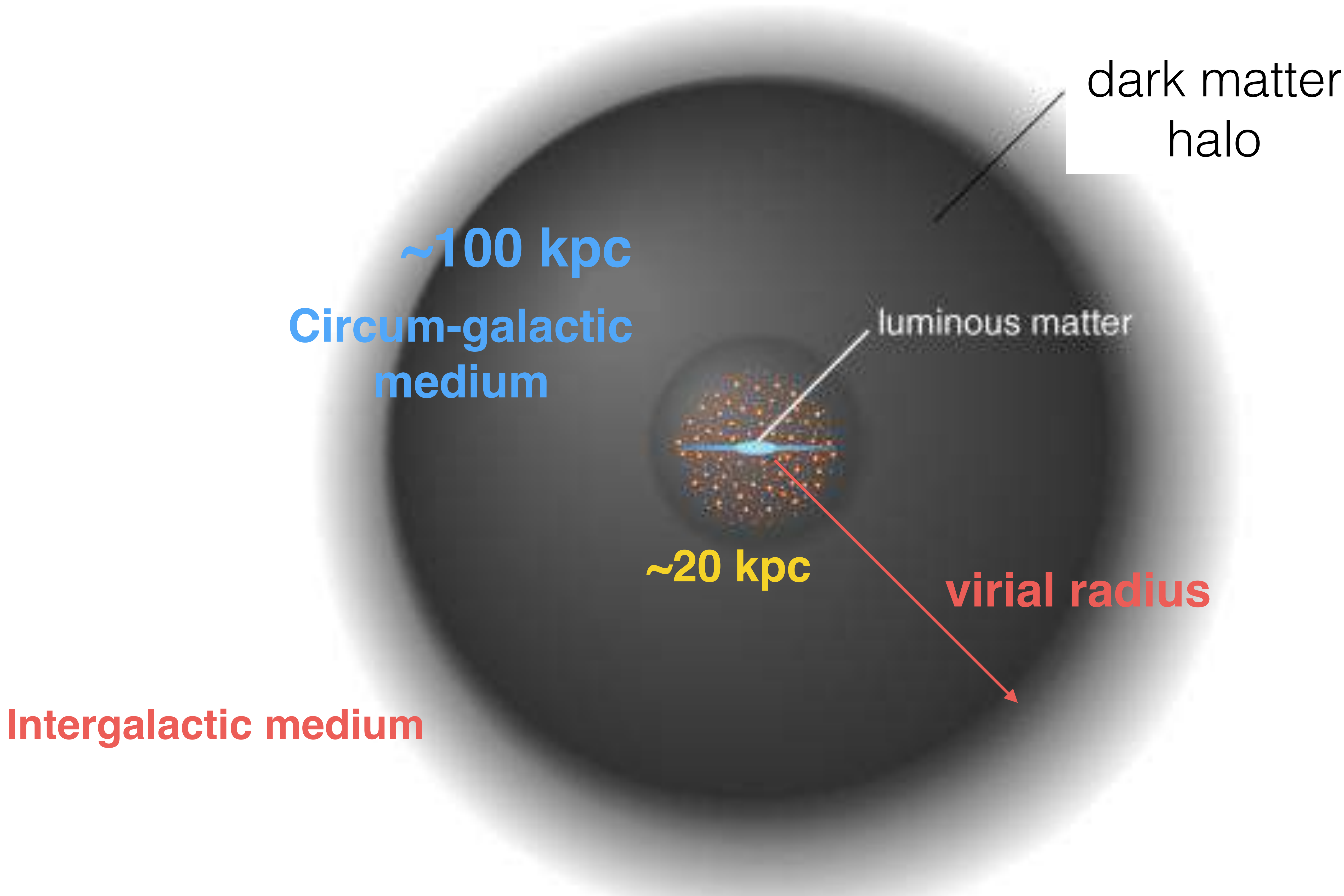
$$g(z) \approx \frac{5}{2}\Omega_m(z)\left\{\Omega_m^{4/7}(z) - \Omega_\Lambda(z) + [1+\Omega_m(z)/2][1+\Omega_\Lambda(z)/70]\right\}^{-1},$$

Carroll+ ‘92

# Linear Growth Rate



# DM halo & galaxy




(cf. Spherical collapse model)

# Press-Schechter Mass Function (1974)

**Ansatz:**

Probability that $\delta_s > \delta_c(t)$ **=** fraction of mass contained in halos with mass >M

$$\boxed{\mathscr{P}[>\delta_c(t)] = \frac{1}{\sqrt{2\pi}\sigma(M)} \int_{\delta_c(t)}^{\infty} \exp\left[-\frac{\delta_s^2}{2\sigma^2(M)}\right] d\delta_s = \frac{1}{2}\,\mathrm{erfc}\left[\frac{\delta_c(t)}{\sqrt{2}\sigma(M)}\right].}$$

mass variance: $\sigma^2(M) = \langle \delta_s^2(\mathbf{x};R)\rangle = \frac{1}{2\pi^2}\int_0^\infty P(k)\widetilde{W}^2(\mathbf{k}R)k^2\,dk$

The mass fraction: $F(>M) = \underbrace{2}_{\text{fudge factor}}\mathscr{P}[>\delta_c(t)].$

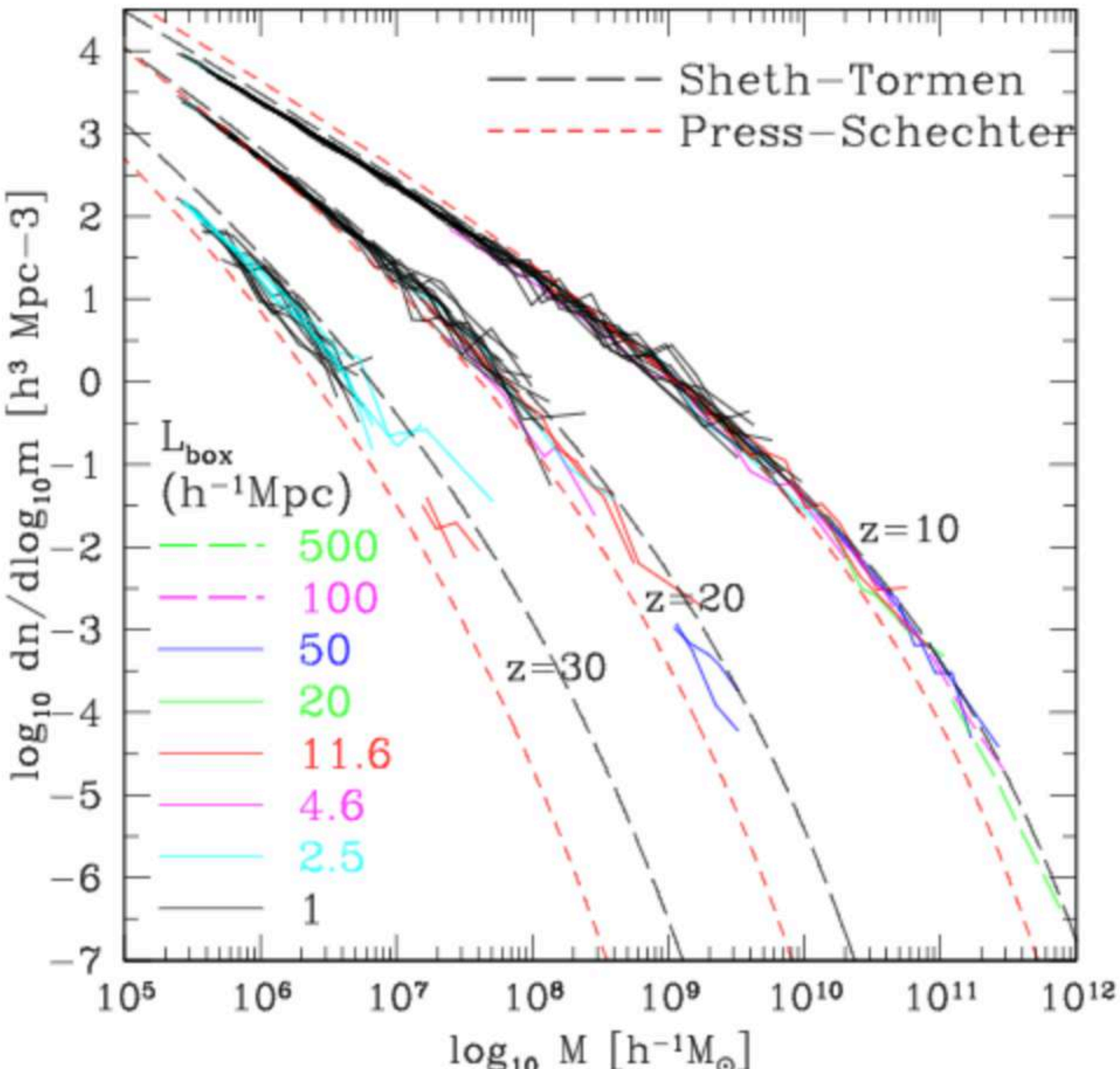
As $M \to 0$,
$\mathscr{P}[>\delta_c(t)] \to 1/2.$

**PS mass function:** $n(M,t)\,dM = \frac{\bar{\rho}}{M}\frac{\partial F(>M)}{\partial M}dM = 2\frac{\bar{\rho}}{M}\frac{\partial \mathscr{P}[>\delta_c(t)]}{\partial \sigma}\left|\frac{d\sigma}{dM}\right| dM$

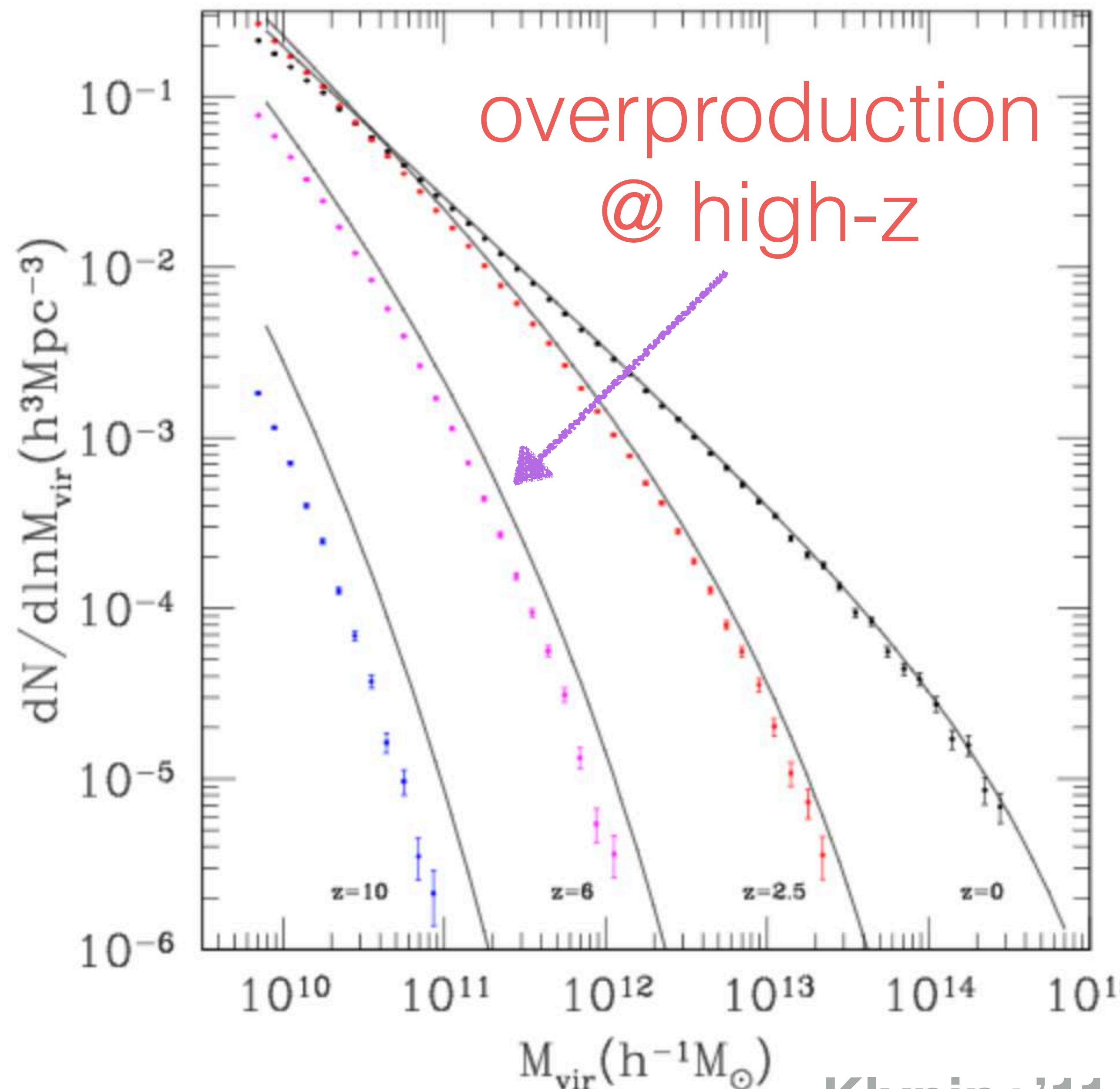
$$= \sqrt{\frac{2}{\pi}}\frac{\bar{\rho}}{M^2}\frac{\delta_c}{\sigma}\exp\left(-\frac{\delta_c^2}{2\sigma^2}\right)\left|\frac{d\ln\sigma}{d\ln M}\right| dM.$$

**Or,** $\boxed{n(M_h,z)dM_h = \sqrt{\frac{2}{\pi}}\frac{\bar{\rho}}{M_h^2}\nu e^{-\nu^2/2}\left|\frac{d\ln\nu}{d\ln M_h}\right| dM_h,}$ $\nu = \delta_c(t)/\sigma(M)$

# Comparison with N-body simulation




Reed+ '07




Klypin+'11

$$\boxed{n(M,z)\,\mathrm{d}M = A\left(1+\frac{1}{\nu'^{2q}}\right)\sqrt{\frac{2}{\pi}}\,\frac{\bar{\rho}}{M}\,\frac{\mathrm{d}\nu'}{\mathrm{d}M}\exp\left(-\frac{\nu'^2}{2}\right)\mathrm{d}M,}$$

(Sheth & Tormen '99)

$$\nu' = \sqrt{a}\,\nu,\ \ a = 0.707,\ A \approx 0.322\ \text{ and }\ q = 0.3.$$

(see also Mo & White '02)

# Dark matter halo cusp

**Navarro-Frenk-White (NFW) profile** (NFW ’96)

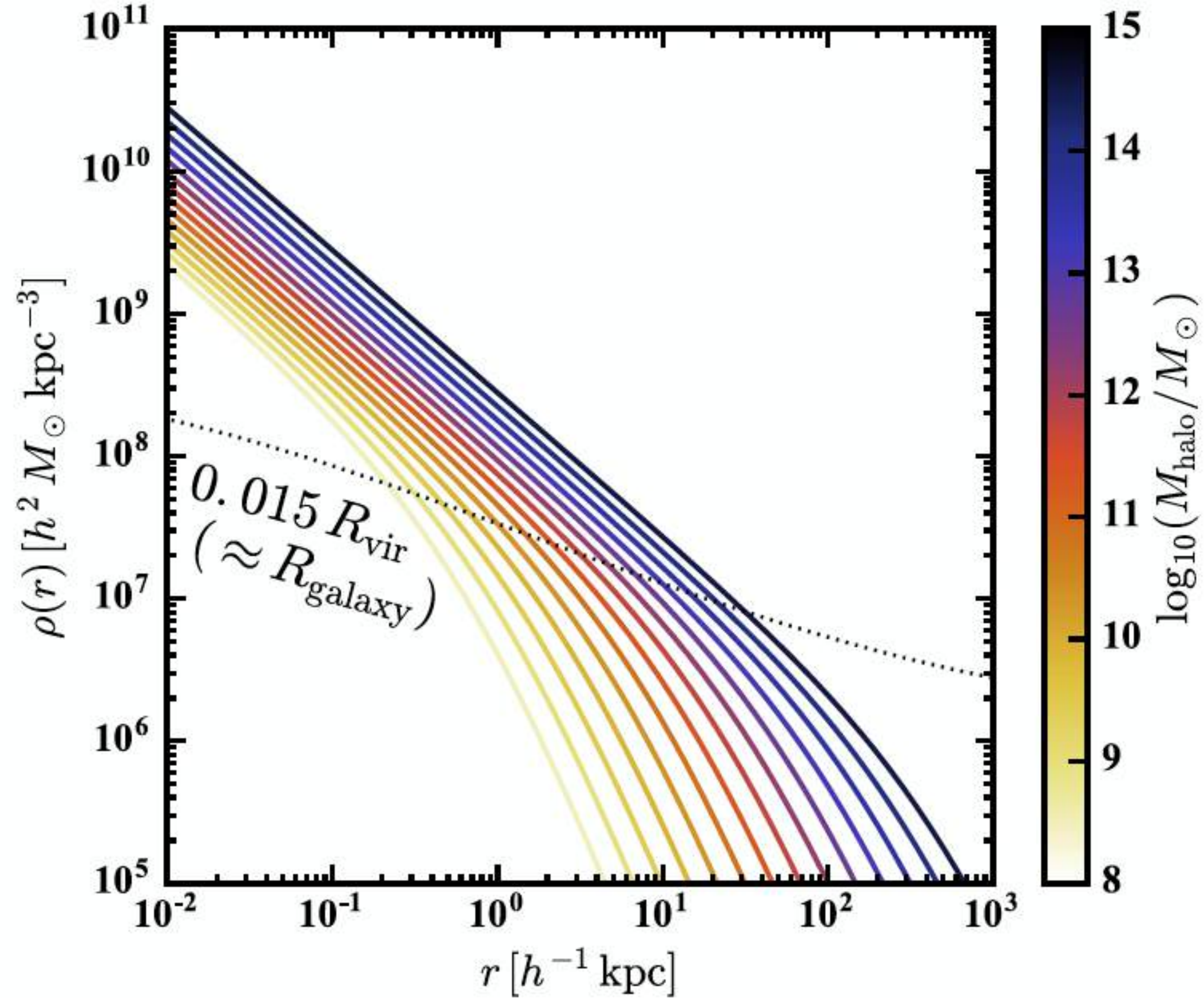
$$\rho(r) = \frac{4\rho_{-2}}{(r/r_{-2})(1+r/r_{-2})^2}.$$

$$c = R_{\rm vir}/r_{-2}.$$

$$c \propto M_{\rm vir}^{-a}\,(1+z)^{-1}$$

a≃0.1 (Bullock+’01)

(see also Klypin+’16-)



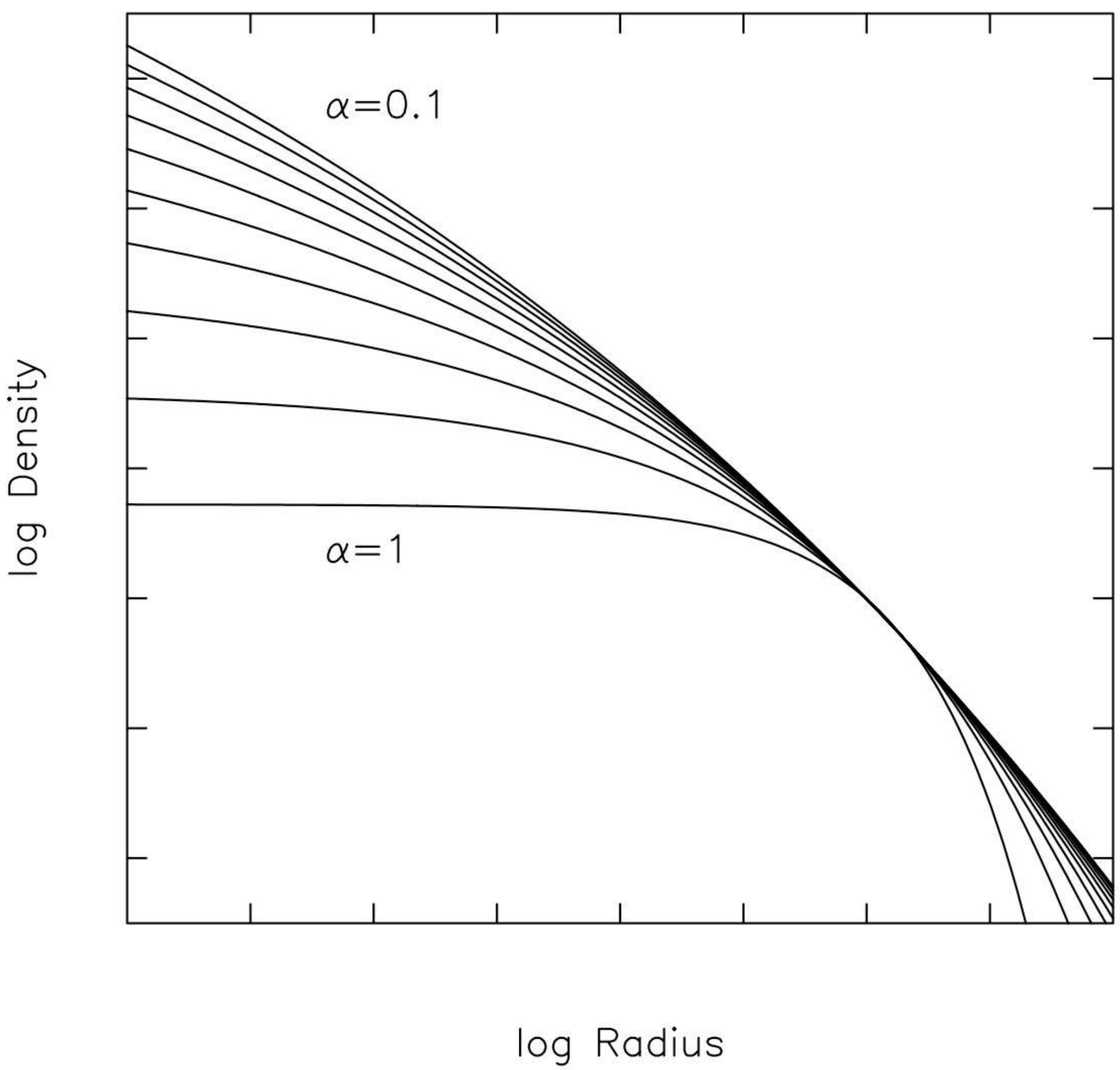

review by Bullock & Boylan-Kolchin ‘17

**But, some observed dwarf gals have flat cores.**

# Einasto Profile

$$\rho_{\rm Ein}(r) = \rho_0 \exp\left(-\frac{2}{\alpha}\left[x^{\alpha}-1\right]\right), \quad x \equiv \frac{r}{r_{-2}}.$$

**Generalization of power-law: $r^{\alpha}$**




$$\gamma(r) \equiv -\frac{\mathrm{d}\ln\rho(r)}{\mathrm{d}\ln r} \propto r^{\alpha}$$

$$\rho(r) \propto \exp\left(-Ar^{\alpha}\right).$$

$$d\left(\log\rho\right)/d\left(\log r\right) \propto -r^{\alpha}.$$

Einasto '63, '65

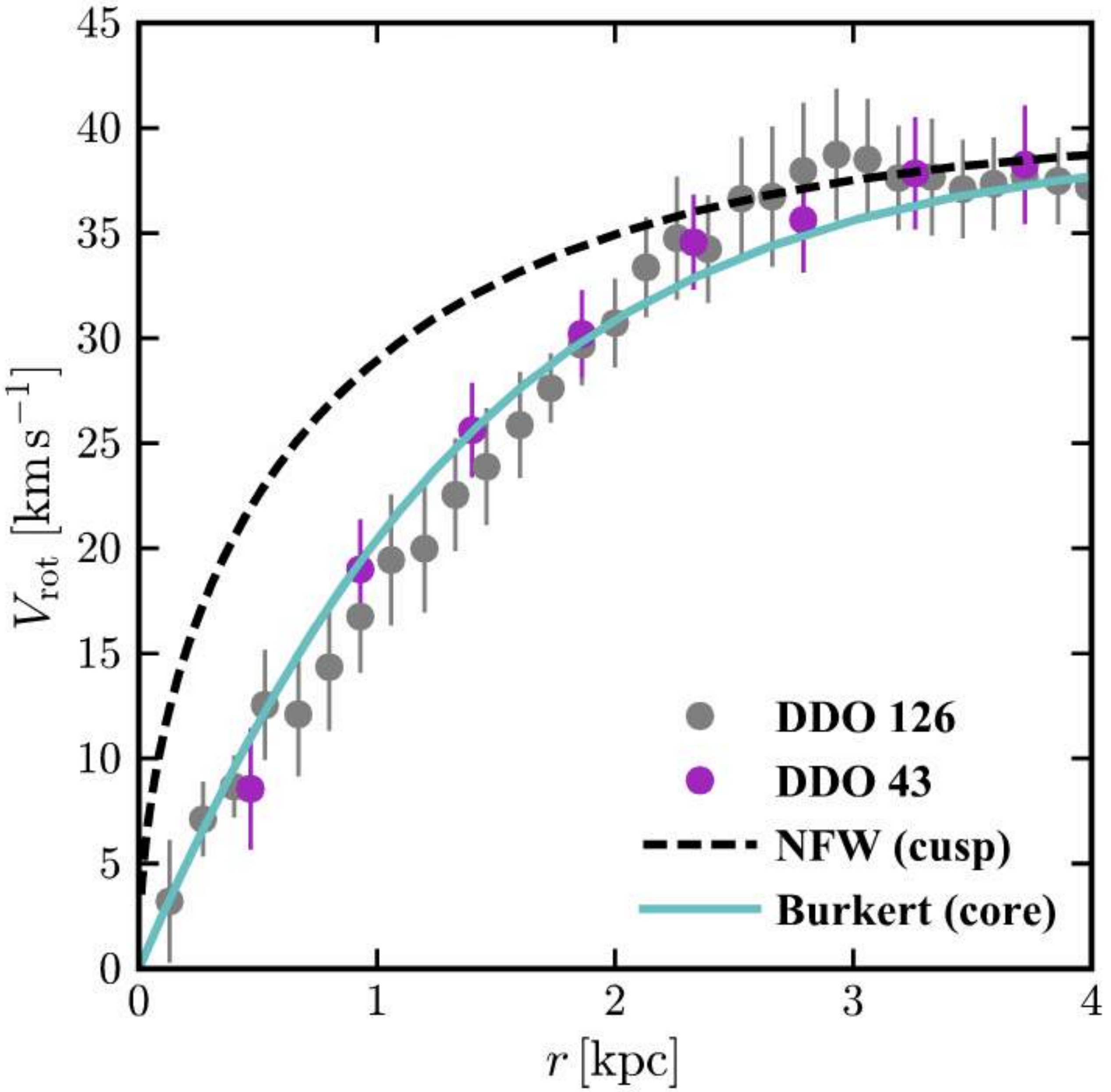
Merritt+ '05, '06

NFW: $$\rho(r) = \frac{\rho_{crit}\delta_c}{(r/r_s)(1+r/r_s)^2}, \qquad \delta_c = \frac{200}{3}\frac{c_h^3}{[\ln(1+c_h) - c_h/(1+c_h)]},$$

concentration param: $c_h = R_{vir}/r_s$

Burkert profile:

$$\boxed{\rho(r) = \frac{\rho_0 r_0^3}{(r+r_s)(r^2+r_s^2)},}$$




LITTLE THINGS survey

(Oh+'15)

# “Recipe” for Galaxy Formation

- ☑ Background Cosmology
- ☑ Gravitational Instability + spherical collapse model
- ☑ N-body dynamics (Dark Matter) — Ch.2 of my book
- ☐ Hydrodynamics
- ☐ Radiative Cooling of Gas, UVB
- ☐ Star Formation, Chemical Enrichment
- ☐ Feedback (SNe, AGNs)

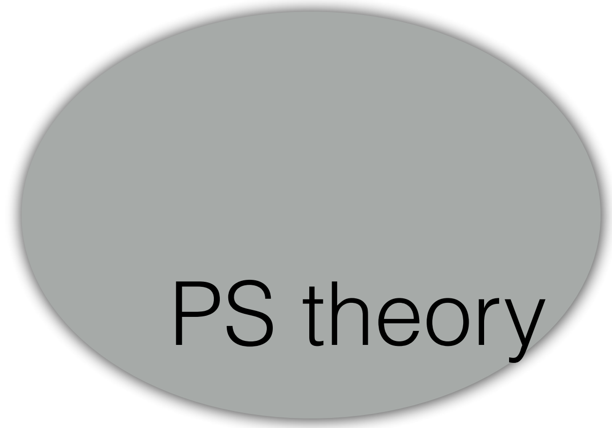
# “1st-order” Galaxy Formation

DM halo forms.

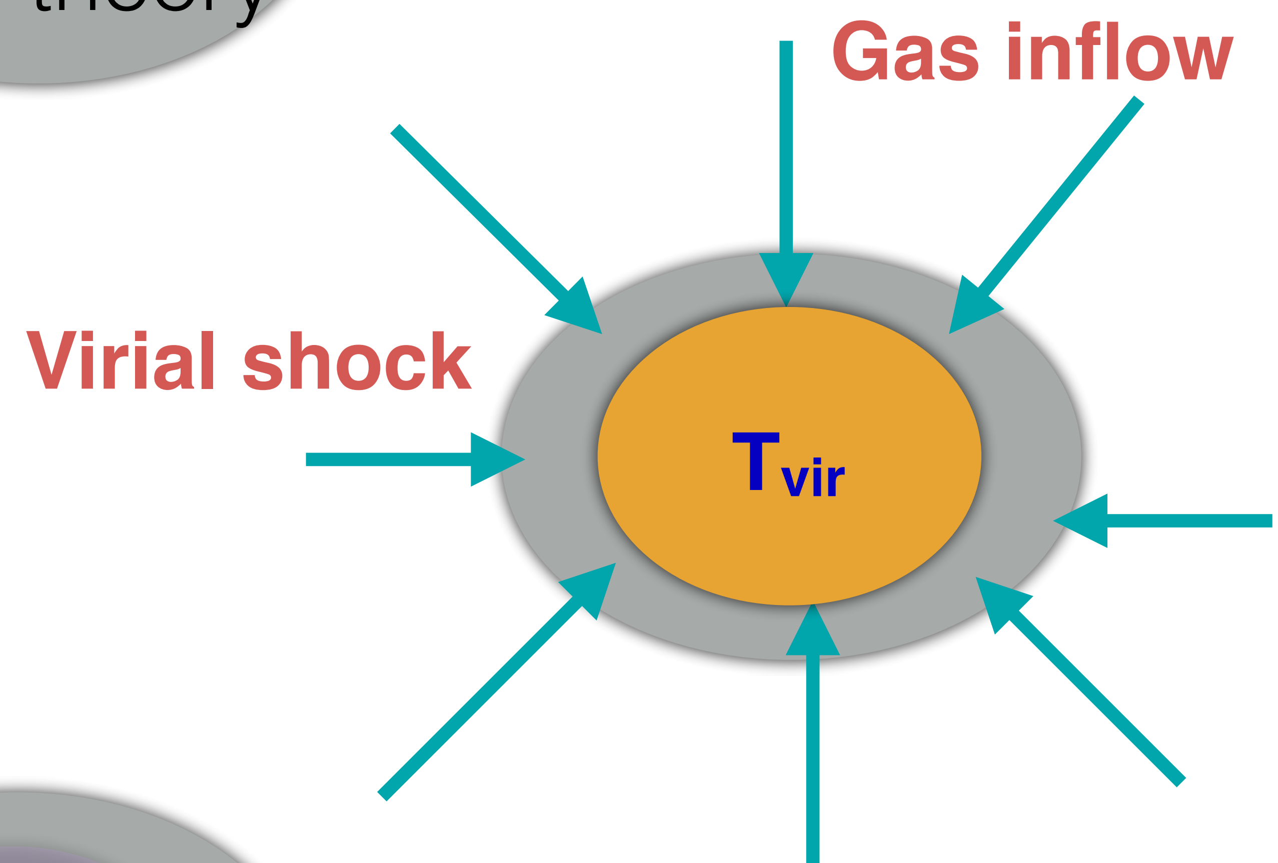
Gas infall
& shock heating

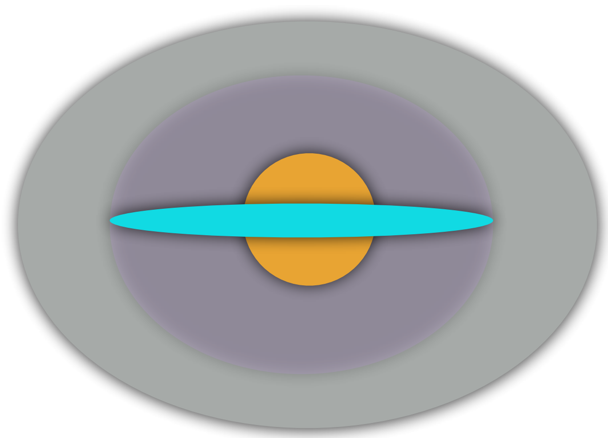
Gas dissipates,
cools,
& forms a disk.

Star, Galaxy Formation




“spherical cow”



$$R_d = \frac{1}{\sqrt{2}}\left(\frac{j_d}{m_d}\right)\lambda r_{200}$$

Rees & Ostriker ’77
White & Rees ’78
Fall & Efstathiou ’80
White & Frenk ‘91
Mo, Mao & White ‘98

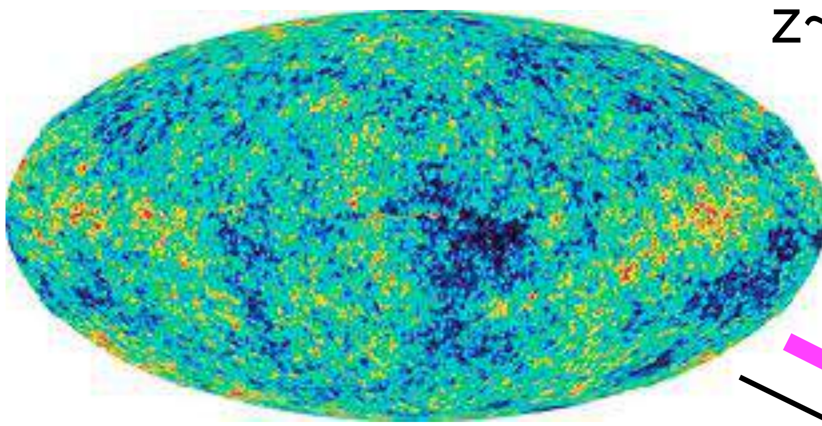
# Part III:
# Complex Baryonic Physics
# (gastro-physics)

**Needs to be studied by numerical simulations (N-body + hydrodynamics) to fully non-linear regime**

# "2nd-order" Galaxy Formation

# Computational Cosmology

Self-consistent galaxy formation scenario from first principles (as much as possible)



z~1100

Initial conditions

Cosmological params,
Dark energy, Dark matter,
Baryons
(+expanding universe)

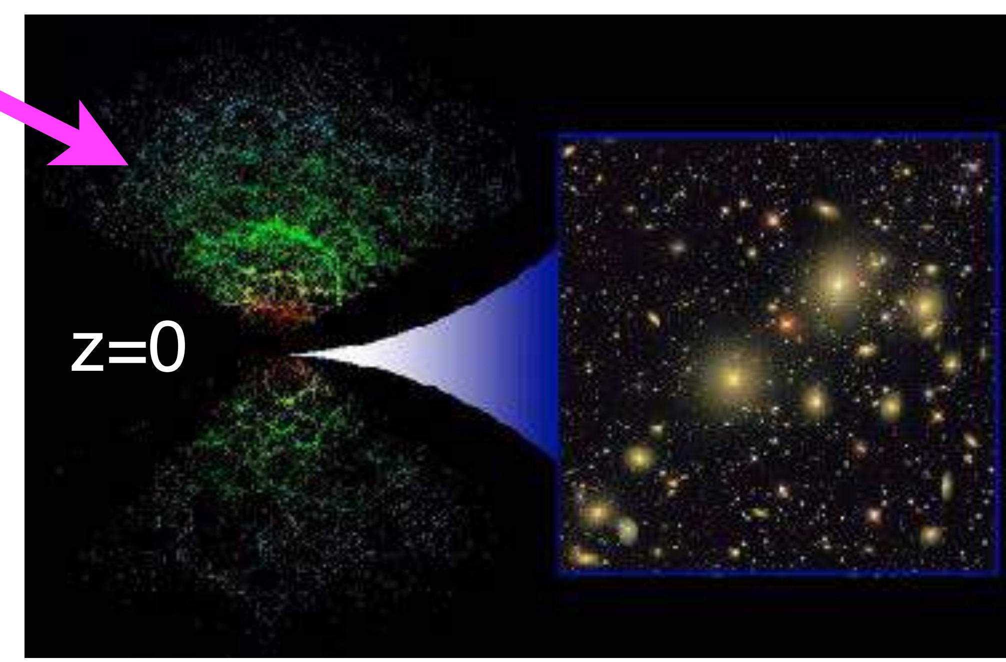
z=100

z=10

z=3

Baryonic physics

Radiative heating/cooling,
Star formation,
& Feedback




Gravity + Hydrodynamics

# Three Revolutions in Cosmological Hydro Simulations

| 1990': 1st Revolution | 2001-2011 2nd Rev. | 2012~ 3rd Rev. |
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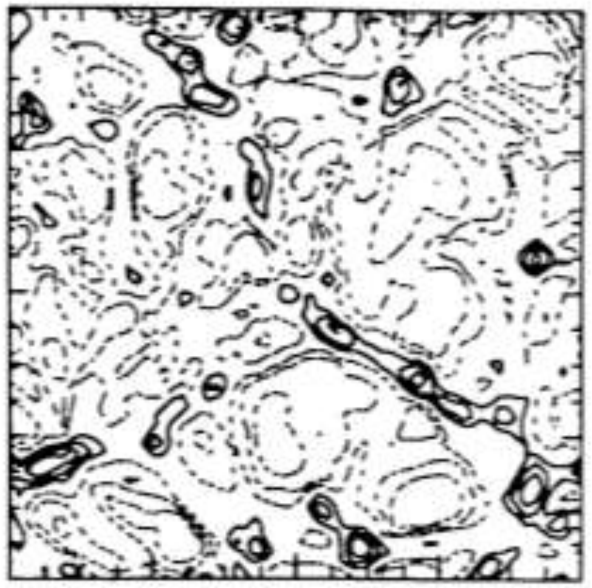
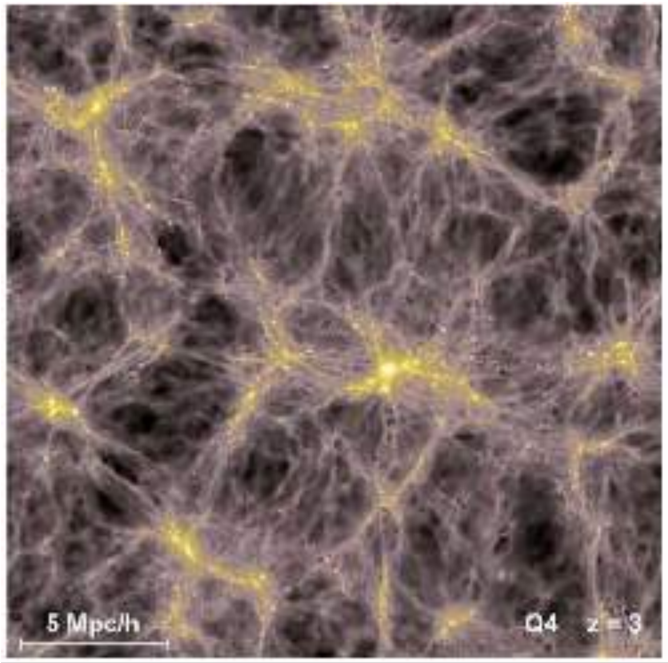
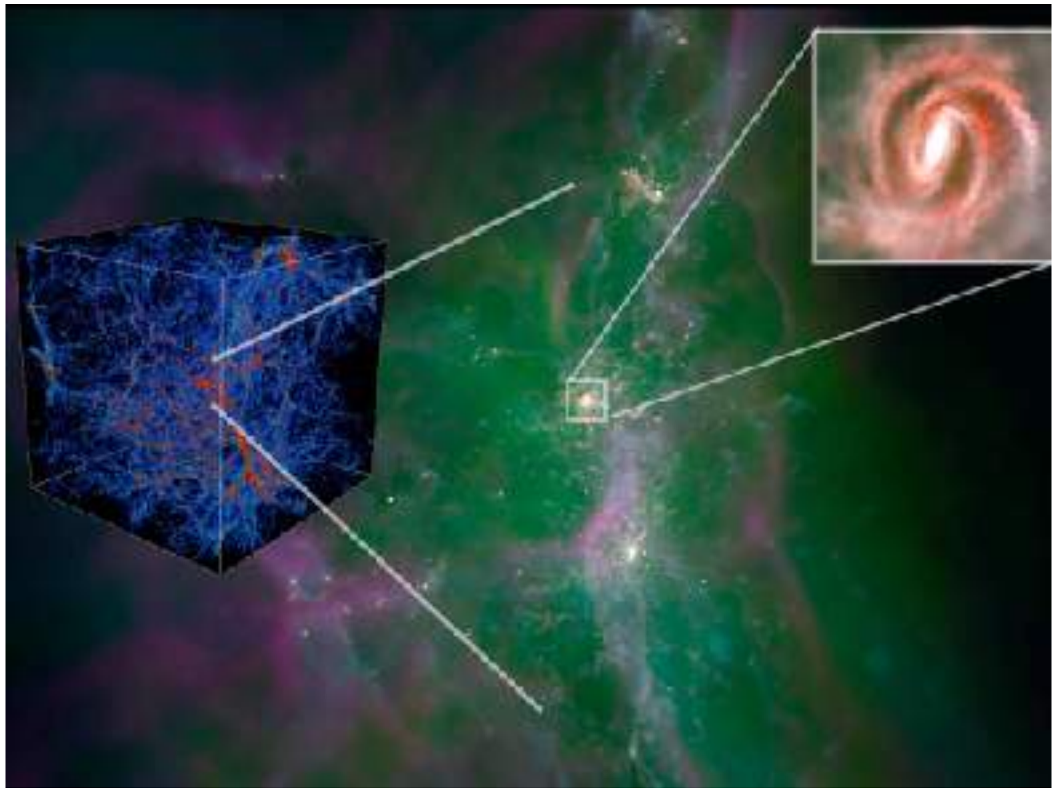
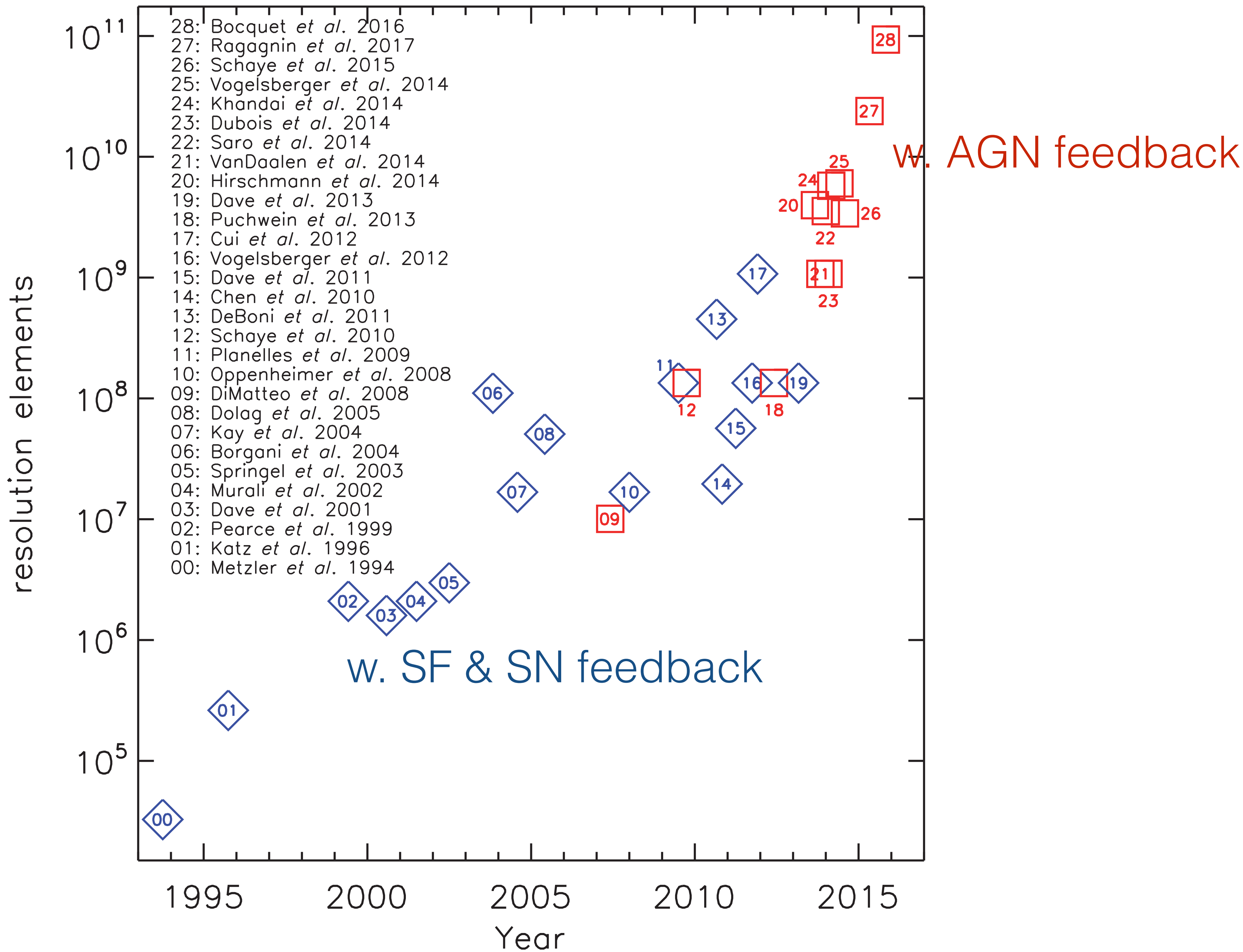
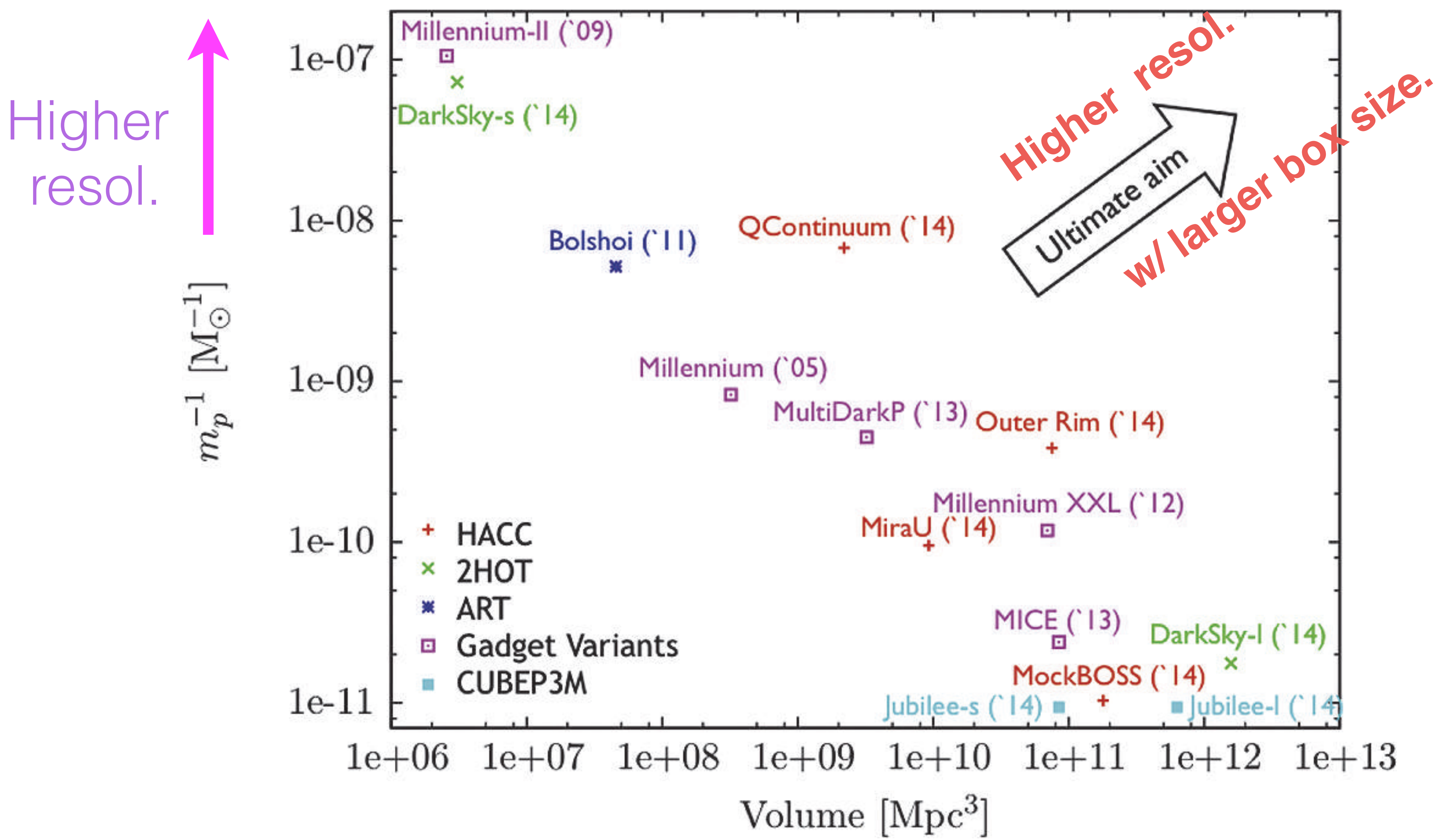
|  |  |  |
| First cosmological, but coarse calculation | Larger scale, medium resolution **w. subgrid models** | Zoom-in method allows much higher res. |
| Resolution~100 kpc | Resolution ~ kpc | Resolution~ 10-100pc |
| e.g. Cen, Ostriker '92-'93<br>Katz+ '96 | e.g. KN+ '01, 04, 06<br>Springel & Hernquist '03 | IC code: GRAFIC (Bertschinger)<br>MUSIC (Hahn & Abel '11) |

# Increasing Resolution Elements




Dolag+

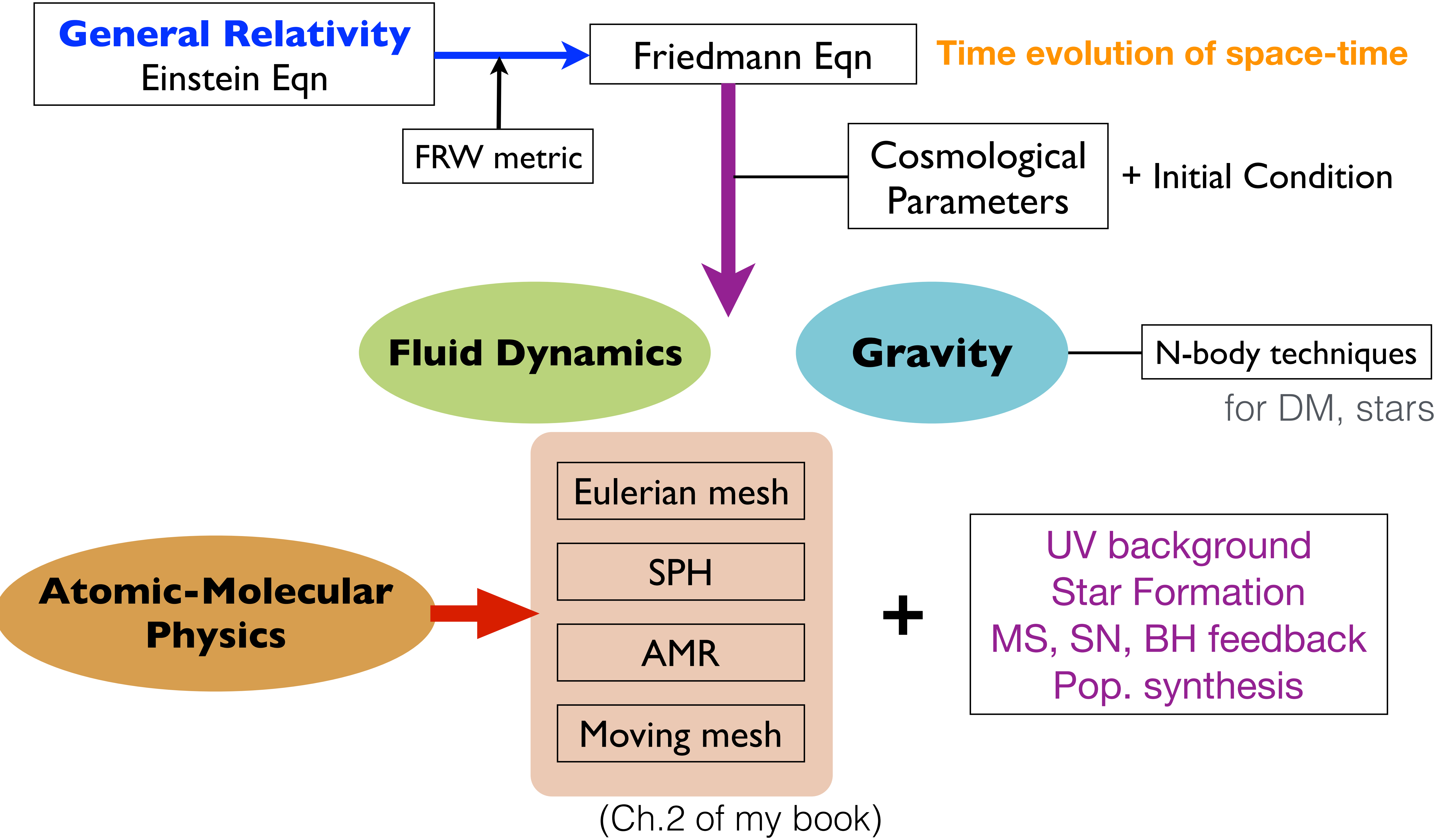
# Mass resolution vs. Box size



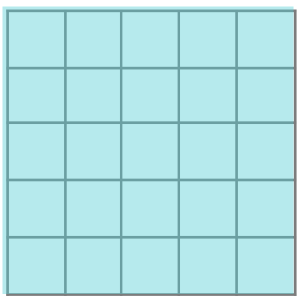

$$m_{\rm DM} = \Omega_{\rm DM}\, \rho_{c,0}\, L_{\rm box}^3 / N_p,$$

Heitmann+'15

# Framework of Computational Cosmology

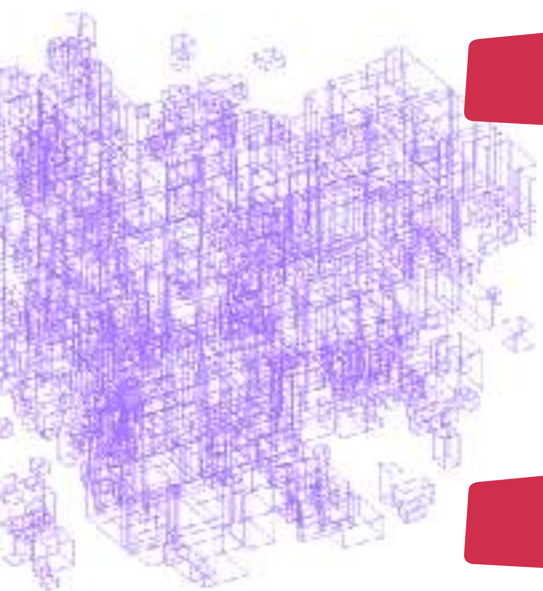


# Cosmological Hydro Codes



## Eulerian mesh (e.g. Cen & Ostriker ’92; KN+’01)

- Eulerian mesh, PM gravity solver, shock capturing hydro
- fast; good baryonic mass resolution at early times
- low final spatial resolution in high-ρ regions, but good at low-ρ regions

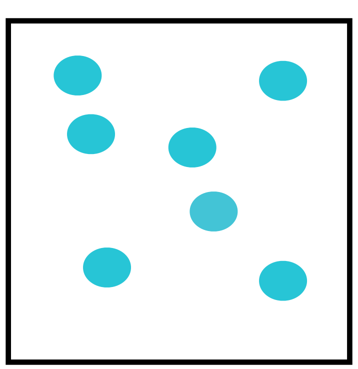


## AMR (adaptive mesh refinement: e.g. Enzo, RAMSES, etc.)

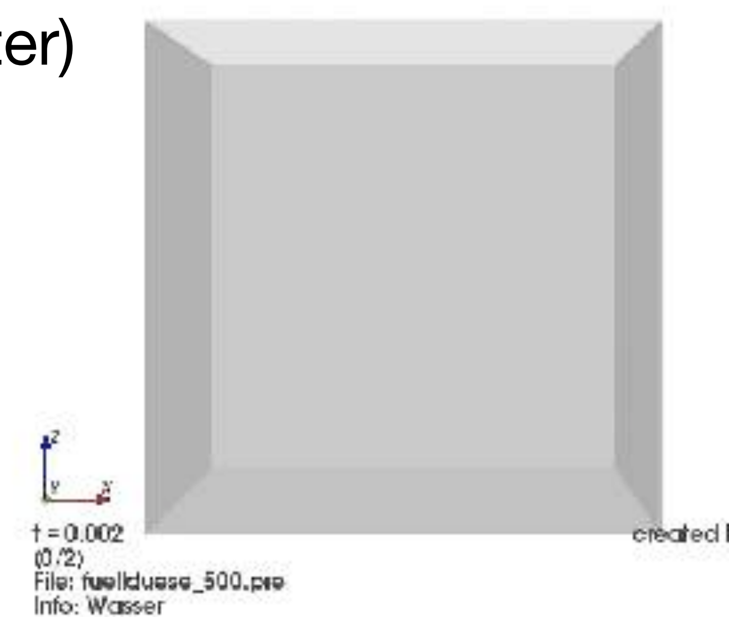
- Eulerian root grid, refine as necessary
- multi-grid PM gravity solver, ZEUS hydro, PPM hydro
- high dynamic range, but slower

**AMR-SPH comparison:**
O’Shea, KN+ ‘05

## SPH (Smoothed Particle Hydrodynamics: e.g. GADGET, GASOLINE, etc.)



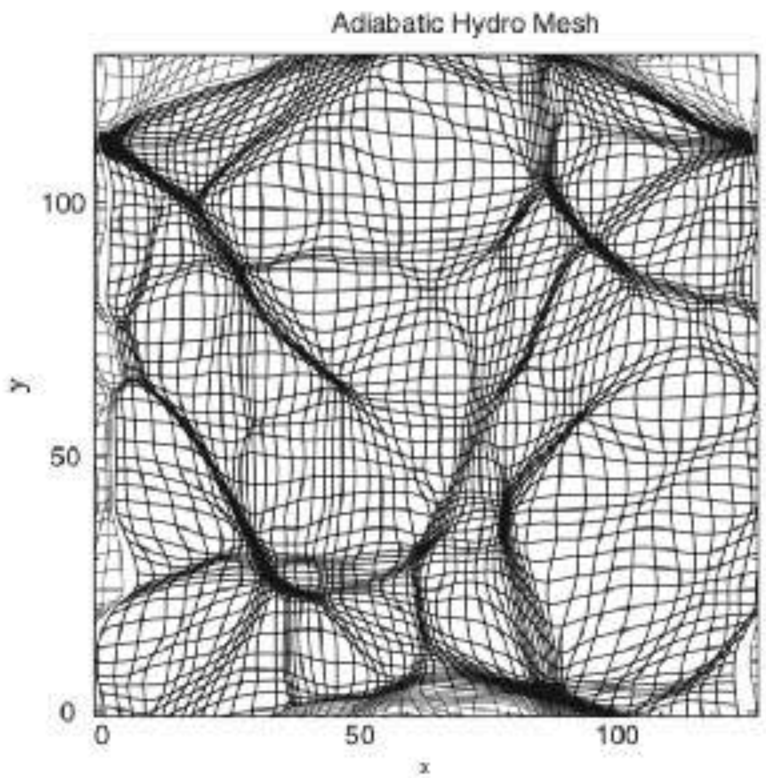
- Lagrangian, particle-based (both gas & dark matter)
- Tree-PM for gravity
- SPH for hydro
- fast; good spatial resolution in high-ρ region, but not so good in low-ρ region



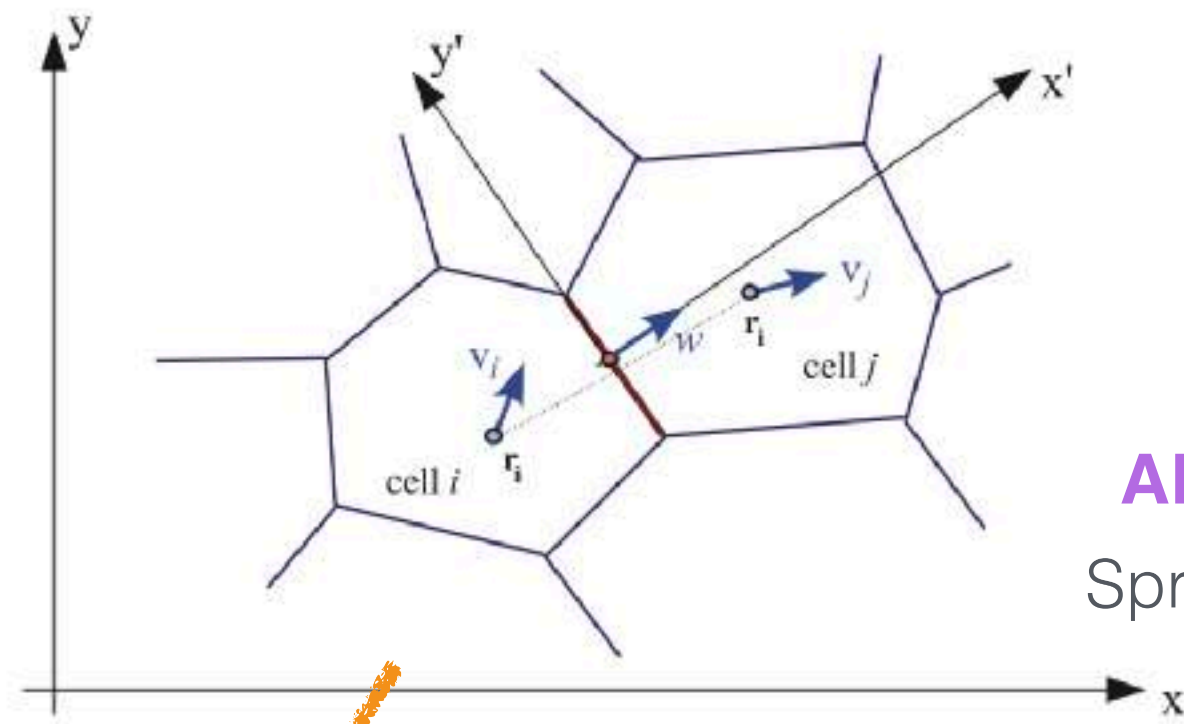
Furthermore,

## Moving mesh method:

Voronoi tesselation




Pen '98




**AREPO**

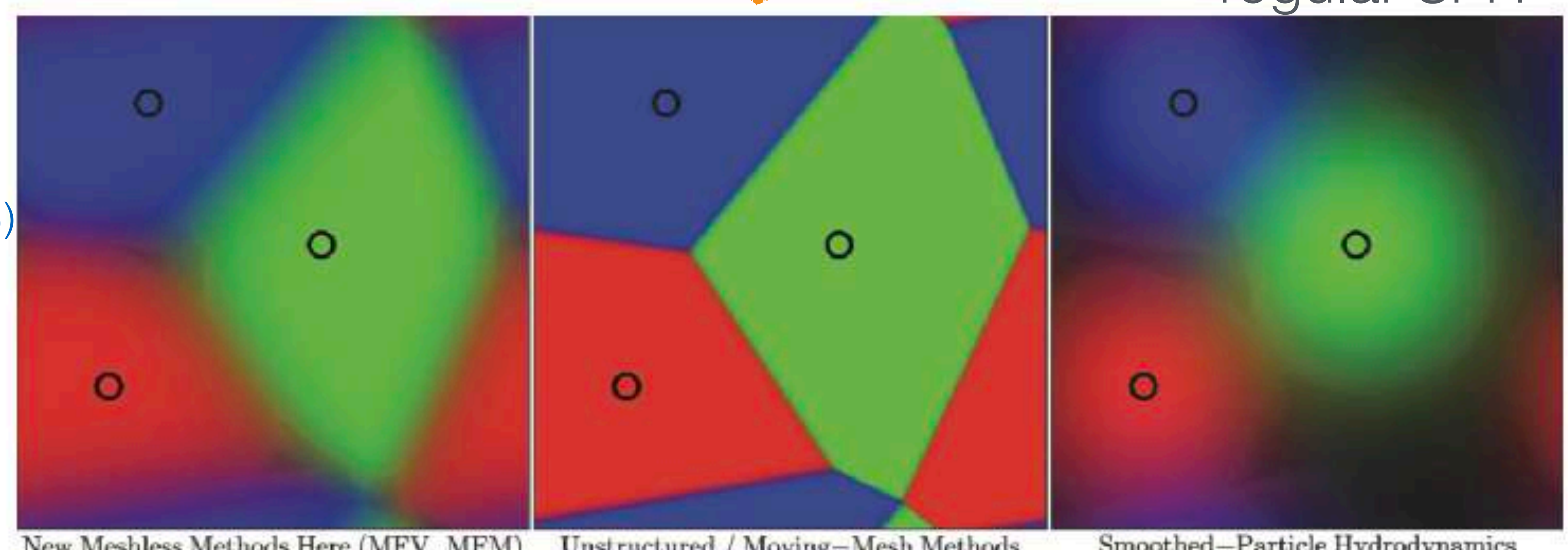
Springel '10

## Mesh-less methods:

regular SPH

**Gizmo**

(based on GADGET-3)




Hopkins '12

# Cosmological Hydrodynamics

Mass consv. $$\frac{\partial \rho}{\partial t} + \frac{1}{a}\frac{\partial}{\partial x_k}(\rho u_k) = 0 \,,$$

$\rho$ : comoving density

$u$ : proper peculiar vel.

Momentum consv. $$\frac{\partial \rho u_i}{\partial t} + \frac{1}{a}\frac{\partial}{\partial x_k}(\rho u_i u_k + P\delta_{ik}) = -\frac{\dot{a}}{a}\rho u_i \,,$$

Energy consv. $$\frac{\partial E}{\partial t} + \frac{1}{a}\frac{\partial}{\partial x_k}[(E + P)u_k] = -\frac{2\dot{a}}{a}E \,.$$

where $E \equiv P/(\gamma - 1) + \frac{1}{2}\rho u_k^2$

total specific energy per comoving vol.

Plus

1. The gravitational source term in the comoving coordinates.
2. The Compton cooling (or heating) term due to interactions of free electrons with the diffuse microwave background radiation field and the diffuse X-ray background radiation field.
3. The integrated radiative cooling-heating term due to hydrogen and helium, which includes electron bremsstrahlung cooling, hydrogen and helium recombination cooling, helium dielectronic recombination cooling, hydrogen and helium collisional ionization cooling, hydrogen and helium collisional excitation cooling, and photoionization heating.
4. Numerical diffusion terms and terms induced from them.

Cen ’92; Cen & Ostriker ’93~; …

H, He, Hii, Heii, ….

$$\frac{\partial\rho}{\partial t}+\frac{1}{a}\frac{\partial}{\partial x_k}(\rho u_k-D_{\rho k})=0\,, \tag{4a}$$

$$\frac{\partial\rho_{\mathrm{H\,I}}}{\partial t}+\frac{1}{a}\frac{\partial}{\partial x_k}(\rho_{\mathrm{H\,I}}u_k-D_{\rho_{\mathrm{H\,I}}k})=n(e)(f_{\mathrm{H}}\rho-\rho_{\mathrm{H\,I}})\alpha_{\mathrm{H\,II}}(T)-n(e)\rho_{\mathrm{H\,I}}\beta_{\mathrm{H}}(T)-\rho_{\mathrm{H\,I}}\int_{\nu_0(\mathrm{H})}^{\infty}4\pi\sigma_\nu(\mathrm{H})\frac{i(\nu)}{h\nu}\,d\nu\,, \tag{4b}$$

$$\begin{aligned}\frac{\partial\rho_{\mathrm{He\,I}}}{\partial t}+\frac{1}{a}\frac{\partial}{\partial x_k}(\rho_{\mathrm{He\,I}}u_k-D_{\rho_{\mathrm{He\,I}}k})&=n(e)\rho_{\mathrm{He\,II}}[\alpha_{\mathrm{He\,II}}(T)+\xi_{\mathrm{He\,II}}(T)]-n(e)\rho_{\mathrm{He\,I}}\beta_{\mathrm{He\,I}}(T)\\&\quad-\rho_{\mathrm{He\,I}}\int_{\nu_0(\mathrm{He\,I})}^{\infty}4\pi\sigma_\nu(\mathrm{He\,I})\frac{i(\nu)}{h\nu}\,d\nu\,,\end{aligned} \tag{4c}$$

$$\begin{aligned}\frac{\partial\rho_{\mathrm{He\,II}}}{\partial t}+\frac{1}{a}\frac{\partial}{\partial x_k}[\rho_{\mathrm{He\,II}}u_k-D_{\rho_{\mathrm{He\,II}}k}]&=n(e)[(1-f_{\mathrm{H}})\rho-\rho_{\mathrm{He\,I}}-\rho_{\mathrm{He\,II}}]\alpha_{\mathrm{He\,III}}(T)-n(e)\rho_{\mathrm{He\,II}}\beta_{\mathrm{He\,II}}(T)\\&\quad-\rho_{\mathrm{He\,II}}\int_{\nu_0(\mathrm{He\,II})}^{\infty}4\pi\sigma_\nu(\mathrm{He\,II})\frac{i(\nu)}{h\nu}\,d\nu-n(e)\rho_{\mathrm{He\,II}}[\alpha_{\mathrm{He\,II}}(T)+\xi_{\mathrm{He\,II}}(T)]\\&\quad+n(e)\rho_{\mathrm{He\,I}}\beta_{\mathrm{He\,I}}(T)+\rho_{\mathrm{He\,I}}\int_{\nu_0(\mathrm{He\,I})}^{\infty}4\pi\sigma_\nu(\mathrm{He\,I})\frac{i(\nu)}{h\nu}\,d\nu\,,\end{aligned} \tag{4d}$$

$$\frac{\partial\rho u_i}{\partial t}+\frac{1}{a}\frac{\partial}{\partial x_k}(\rho u_i u_k+P\delta_{ik}-D_{ik})=-\frac{\dot a}{a}\rho u_i-\frac{1}{a}\rho\frac{\partial\phi}{\partial x_i}+\frac{1}{a}\frac{u_i}{u^2}D_{\rho k}\frac{\partial\phi}{\partial x_k}\,, \tag{5}$$

$$\frac{\partial E}{\partial t}+\frac{1}{a}\frac{\partial}{\partial x_k}[(E+P)u_k-D_{Ek}]=-\frac{2\dot a}{a}E-\frac{1}{a}\rho u_k\frac{\partial\phi}{\partial x_k}+\frac{1}{a}D_{\rho k}\frac{\partial\phi}{\partial x_k}-\Lambda_{\mathrm{net}}\,, \tag{6}$$

radiation

$$\frac{\partial i(\nu)}{\partial t}=\frac{\dot a}{a}\left[\nu\frac{\partial i(\nu)}{\partial\nu}-3i(\nu)\right]+c[j_{\mathrm{ff}}(\nu)+j_{\mathrm{fb,H\,II}}+j_{\mathrm{fb,He\,II}}+j_{\mathrm{fb,He\,III}}-\kappa(\nu)i(\nu)]\,, \tag{7}$$

EOS

$$P=\frac{1}{m_p}\left(\rho_{\mathrm{H\,I}}+\rho_{\mathrm{H\,II}}+\frac{\rho_{\mathrm{He\,I}}+\rho_{\mathrm{He\,II}}+\rho_{\mathrm{He\,III}}}{4}+\rho_{\mathrm{H\,II}}+\frac{\rho_{\mathrm{He\,II}}}{4}+\frac{\rho_{\mathrm{He\,III}}}{2}\right)kT\,, \tag{8}$$

Cen ’92

# "Recipe" for Galaxy Formation

- [x] Background Cosmology
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- [ ] Radiative Cooling of Gas, UVB
- [ ] Star Formation, Chemical Enrichment
- [ ] Feedback (SNe, AGNs)

# Cooling Processes

- **Compton cooling:** electron scattering off of photon backgrnd

$$(h_P \nu \ll m_e c^2;\ k_B T_e \ll m_e c^2) \quad T_\gamma \ll T_e$$

Cooling rate per unit vol.

$$\boxed{\mathscr{C}_{\rm Comp} = \frac{du_\gamma}{dt} = \frac{4 k_B T_e}{m_e c^2} c \sigma_T n_e a_r T_\gamma^4,}$$

Cooling time: $$t_{\rm Comp} \approx \frac{3 k_B T_e n_e}{\mathscr{C}_{\rm Comp}} = \frac{3 m_e c}{4 \sigma_T a_r T_\gamma^4},$$

$$T_\gamma \approx 2.73(1+z)\,\mathrm{K}.$$

$$t \approx 6.7 \times 10^9 \Omega_{m,0}^{-1/2} h^{-1} (1+z)^{-3/2}\,\mathrm{yr}$$ (EdS Univ.)

$$\boxed{\frac{t_{\rm Comp}}{t} \approx 350 \Omega_{m,0}^{1/2} h (1+z)^{-5/2}.}$$

**C.C. off of CMB is only important at z>6.**

(cf. Inverse Compton scatt. in gal. clusters — **SZ effect**)

- **Bremsstrahlung:**

free-free emissivity

$$\mathscr{C}_{\rm ff} = \int \varepsilon_{\rm ff}(\nu)\, {\rm d}\nu \quad \approx 1.4 \times 10^{-23} T_8^{1/2} \left(\frac{n_{\rm e}}{\rm cm^{-3}}\right)^2 {\rm erg\, s^{-1}\, cm^{-3}},$$

- **Collisional ionization:** K.E. of e⁻ is used for ionization of atoms.

$$\mathscr{C}_a(T) = \frac{g_a}{g_{a+1}} n_{\rm e} n_{a+1} \left(\frac{2\pi m_{\rm e} k_{\rm B} T}{h_{\rm P}^2}\right)^{-3/2} \frac{4\pi}{c^2} \int_{\nu_a}^{\infty} \nu^2 \sigma_{\rm pi}(\nu, a) h_{\rm P}(\nu - \nu_a) \exp\left[-\frac{h_{\rm P}(\nu - \nu_a)}{k_{\rm B} T}\right] {\rm d}\nu,$$

| | |
|---|---|
| $H^0$ | $1.27 \times 10^{-21} T^{1/2} e^{-157809/T} (1 + T_5^{1/2})^{-1} n_{\rm e} n_{\rm H^0}$ |
| $He^0$ | $9.38 \times 10^{-22} T^{1/2} e^{-285335/T} (1 + T_5^{1/2})^{-1} n_{\rm e} n_{\rm He^0}$ |
| $He^+$ | $4.95 \times 10^{-22} T^{1/2} e^{-631515/T} (1 + T_5^{1/2})^{-1} n_{\rm e} n_{\rm He^+}$ |

- **Recombination:** e⁻ combines with ion, emitting a photon —> escape

| | |
|---|---|
| $H^+$ | $2.82 \times 10^{-26} T^{0.3} (1 + 3.54 T_6)^{-1} n_{\rm e} n_{\rm H^+}$ |
| $He^+$ | $1.55 \times 10^{-26} T^{0.3647} n_{\rm e} n_{\rm He^+}$ |
| $He^{++}$ | $1.49 \times 10^{-25} T^{0.3} (1 + 0.885 T_6)^{-1} n_{\rm e} n_{\rm He^{++}}$ |

- **Collisional excitation:** atoms excited by collisions with e⁻ —> decay & emit photon

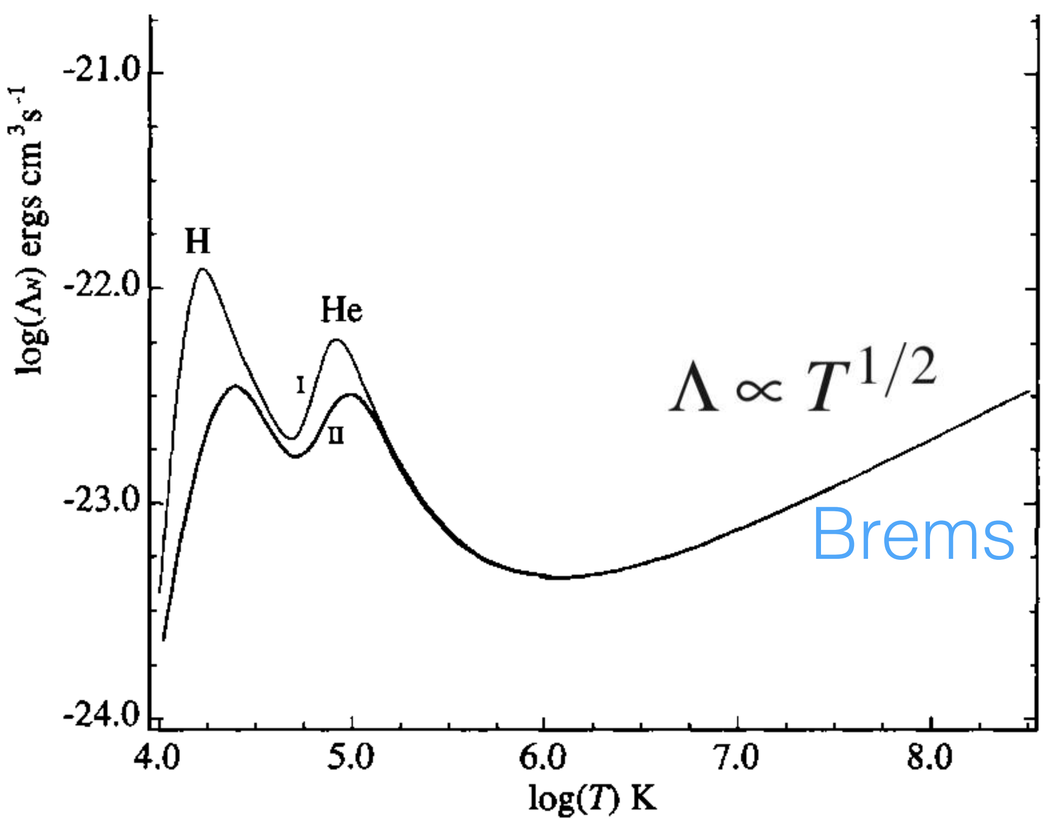
| | |
|---|---|
| $H^0$ | $7.50 \times 10^{-19} e^{-118348/T} (1 + T_5^{1/2})^{-1} n_{\rm e} n_{\rm H^0}$ |
| $He^+$ | $5.54 \times 10^{-17} T^{-0.397} e^{-473638/T} (1 + T_5^{1/2})^{-1} n_{\rm e} n_{\rm He}^{+}$ |

# Cooling Curve

(Radiative Cooling Rate/Function)

**Primordial Gas** — optically thin gas

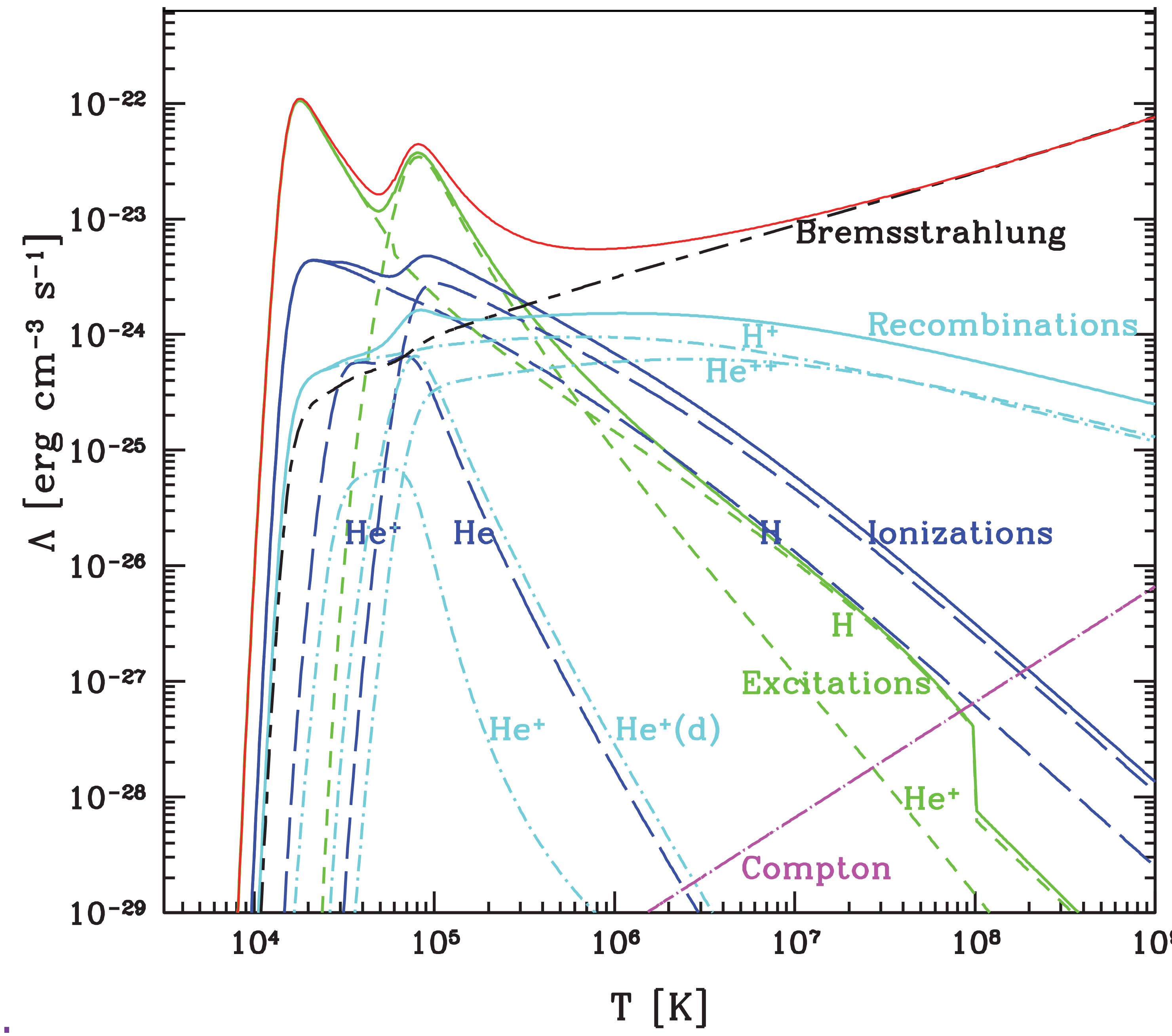
Sutherland & Dopita '93




$$\Lambda(T) \equiv \frac{\mathscr{C}}{n_H^2}, \quad [\mathrm{erg\ cm^3\ s^{-1}}]$$

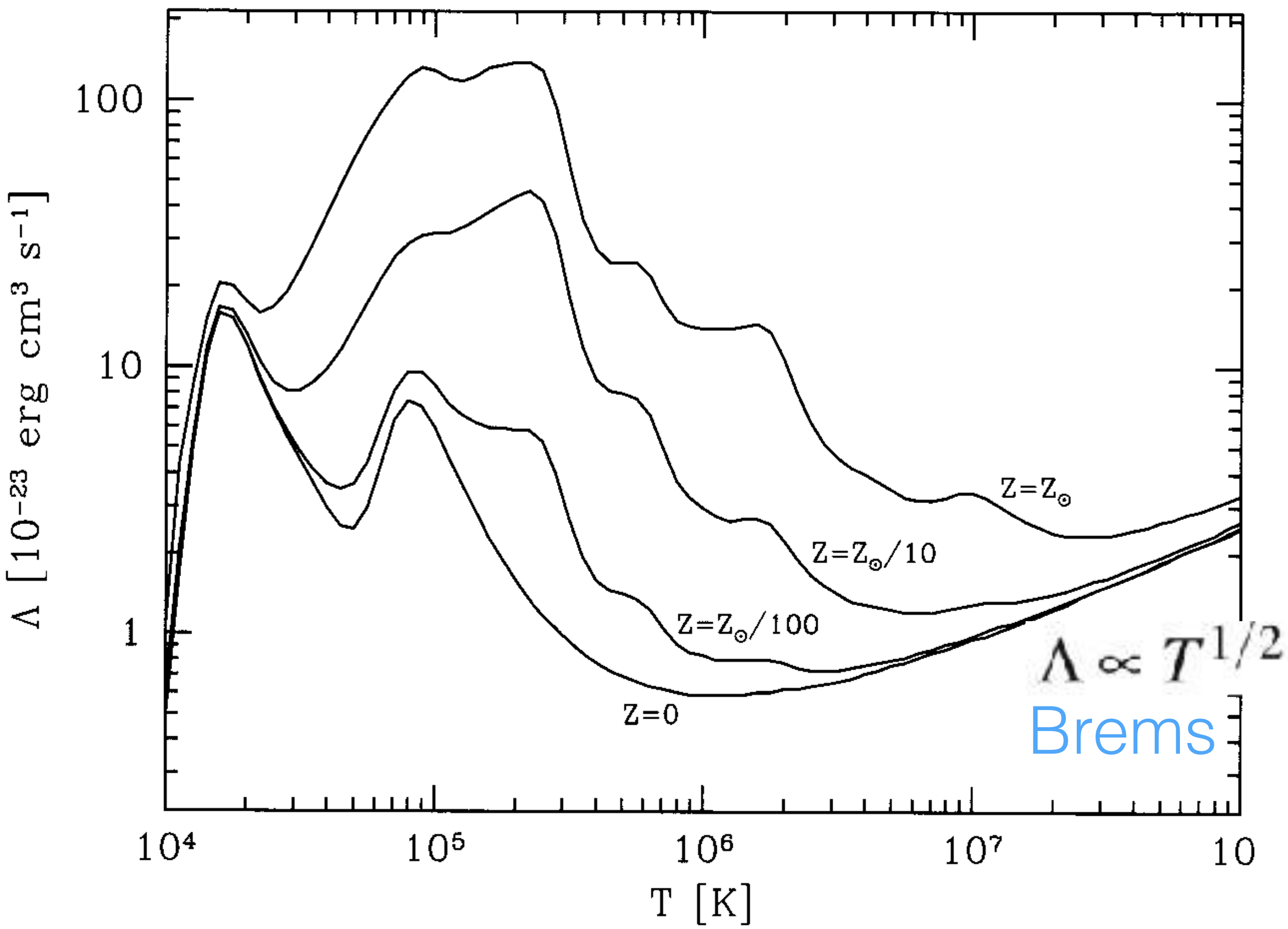
$\mathscr{C}$ : cooling rate per unit vol.

$[\mathrm{erg\ cm^{-3}\ s^{-1}}]$




Cen '92; Katz+'96

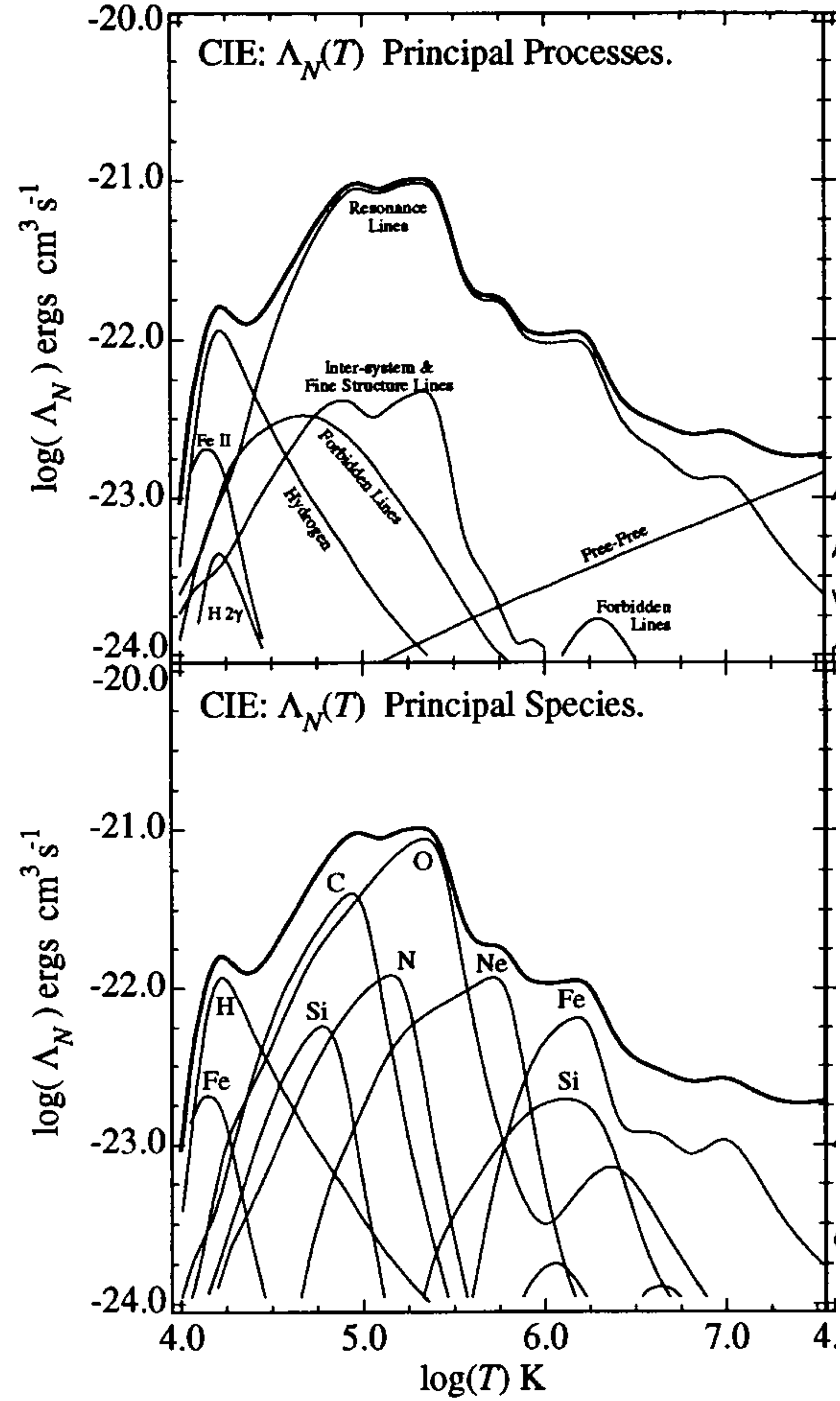
Sutherland & Dopita 1993, ApJS, 88, 253



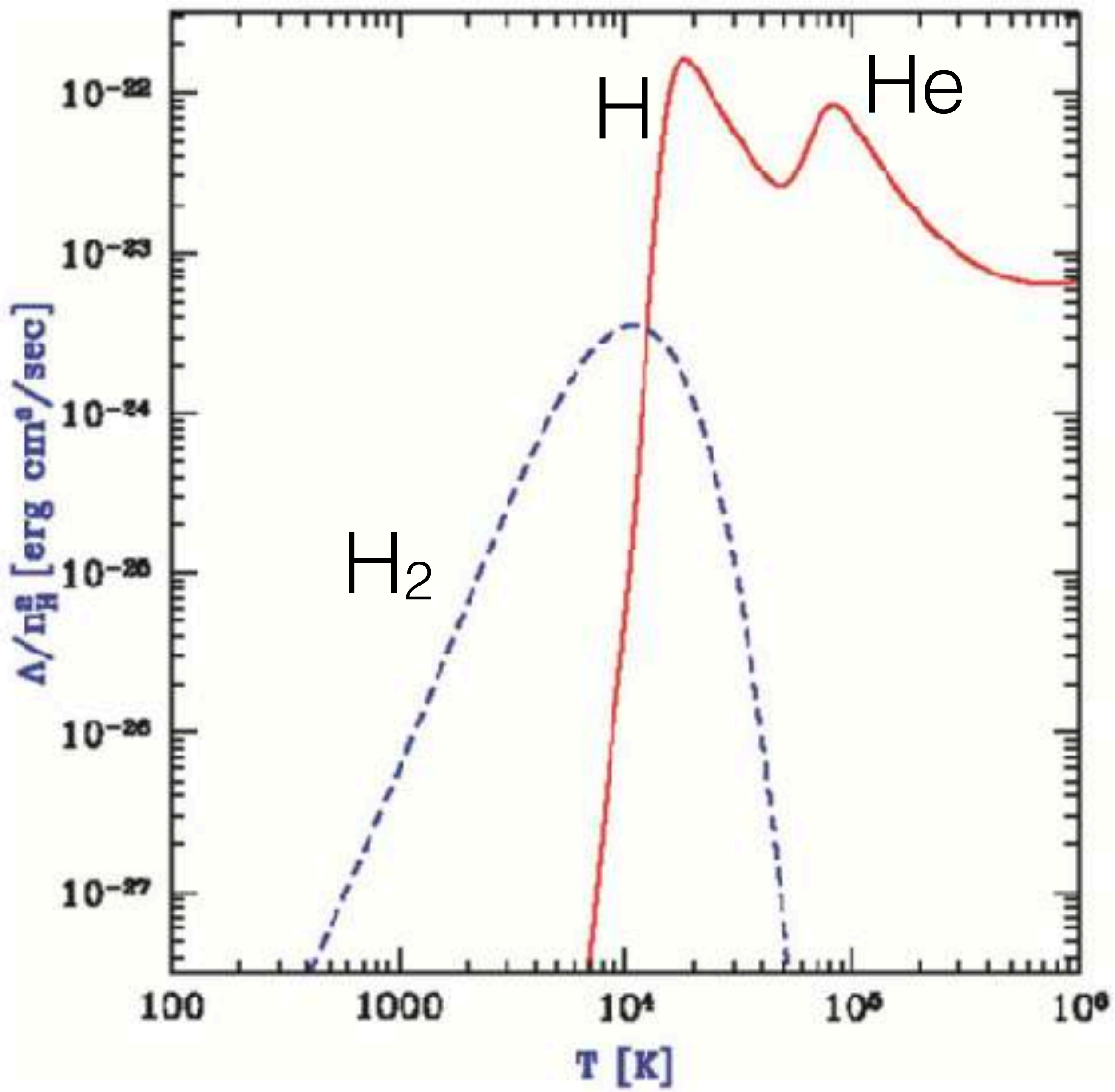

$$\Lambda(T) \equiv \frac{\mathscr{C}}{n_{\mathrm{H}}^2},$$ **[erg cm³ s⁻¹]**

$\mathscr{C}$ : cooling rate per unit vol.

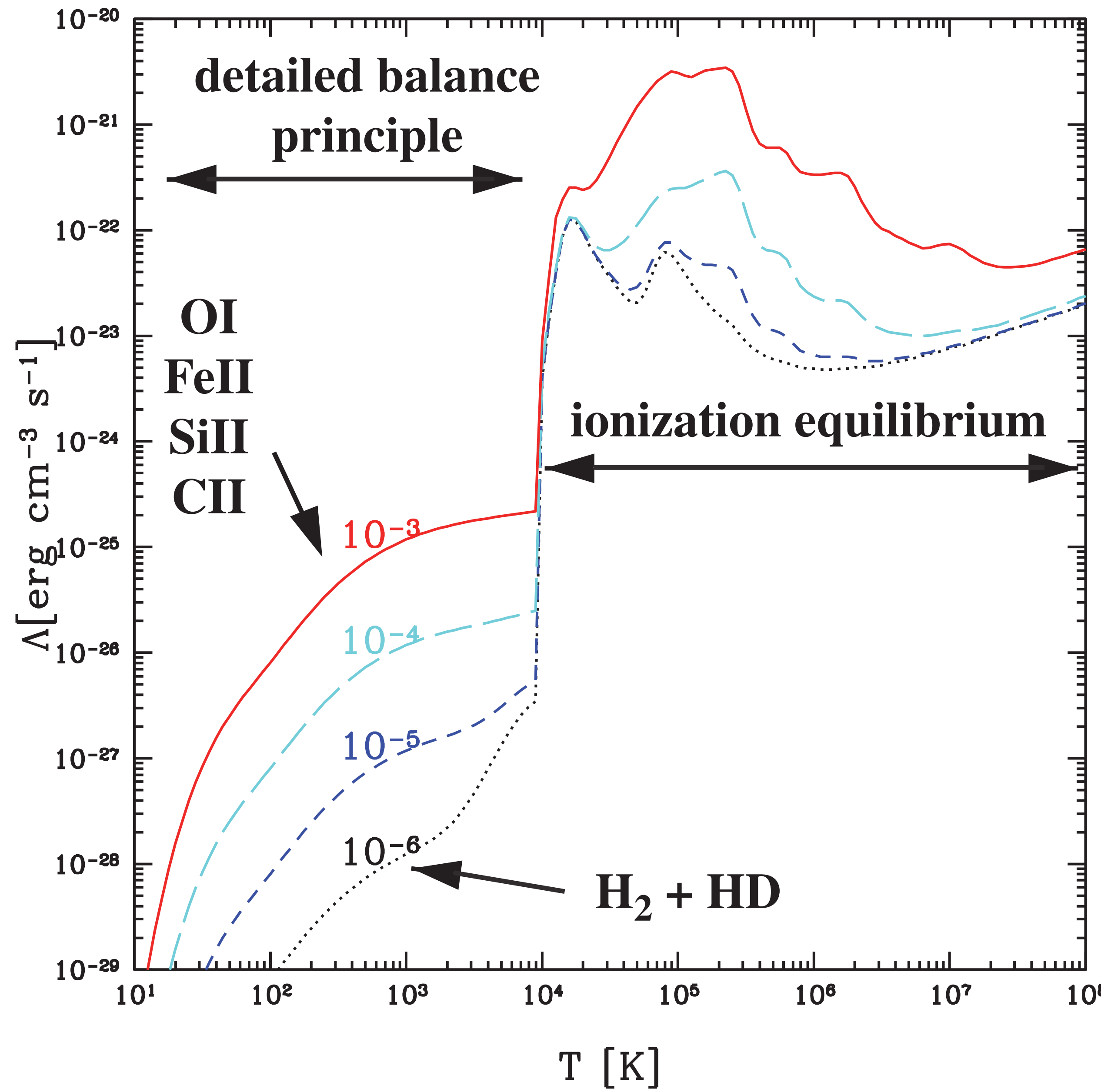
**[erg cm⁻³ s⁻¹]**



# Cooling Curve @ T<10$^4$ K




Barkana & Loeb ‘01

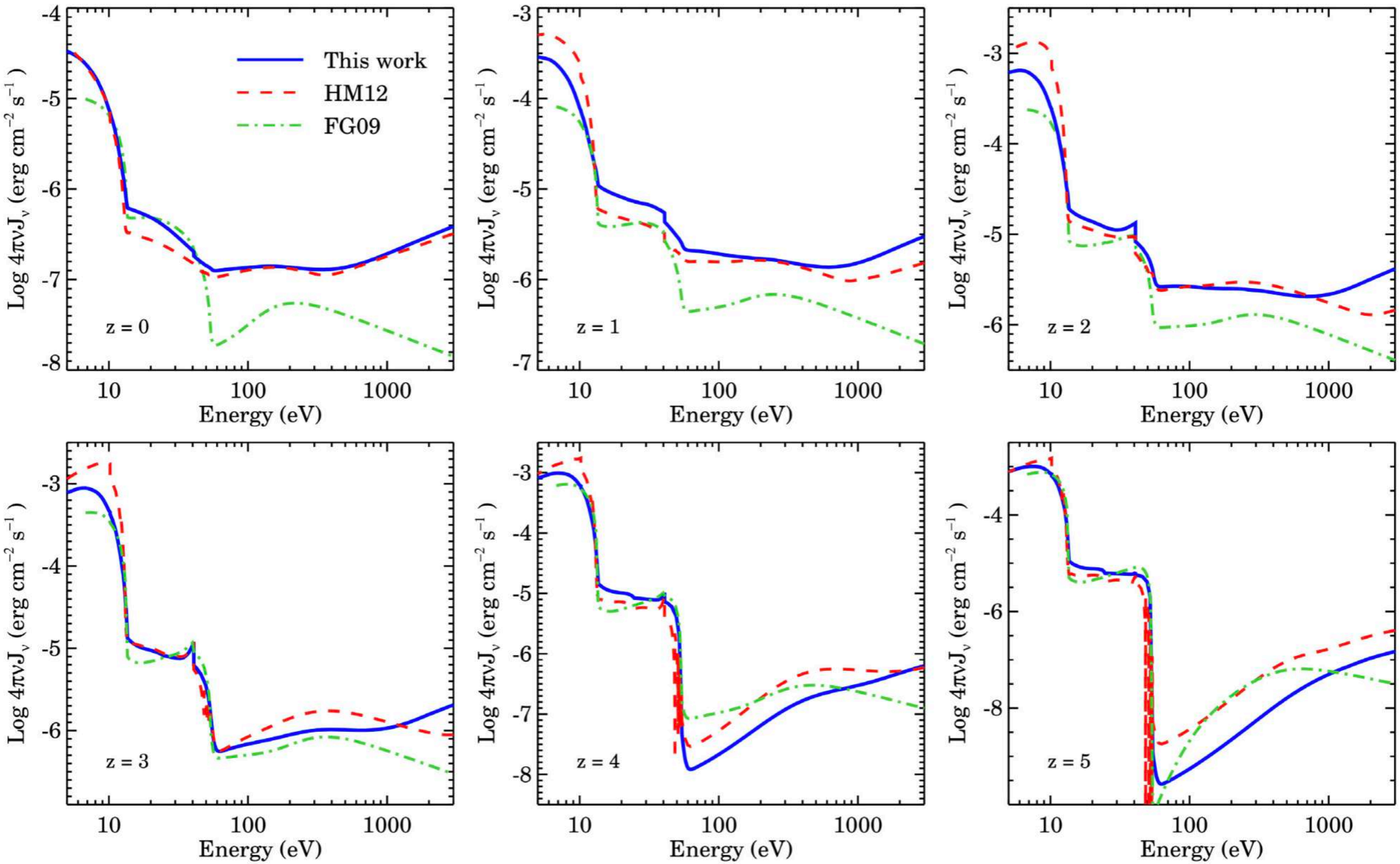



Maio+’07

cf: $T_{vir}$ ~ $10^4$ K for atomic cooling halo of $M_h$~$10^8$ $M_\odot$

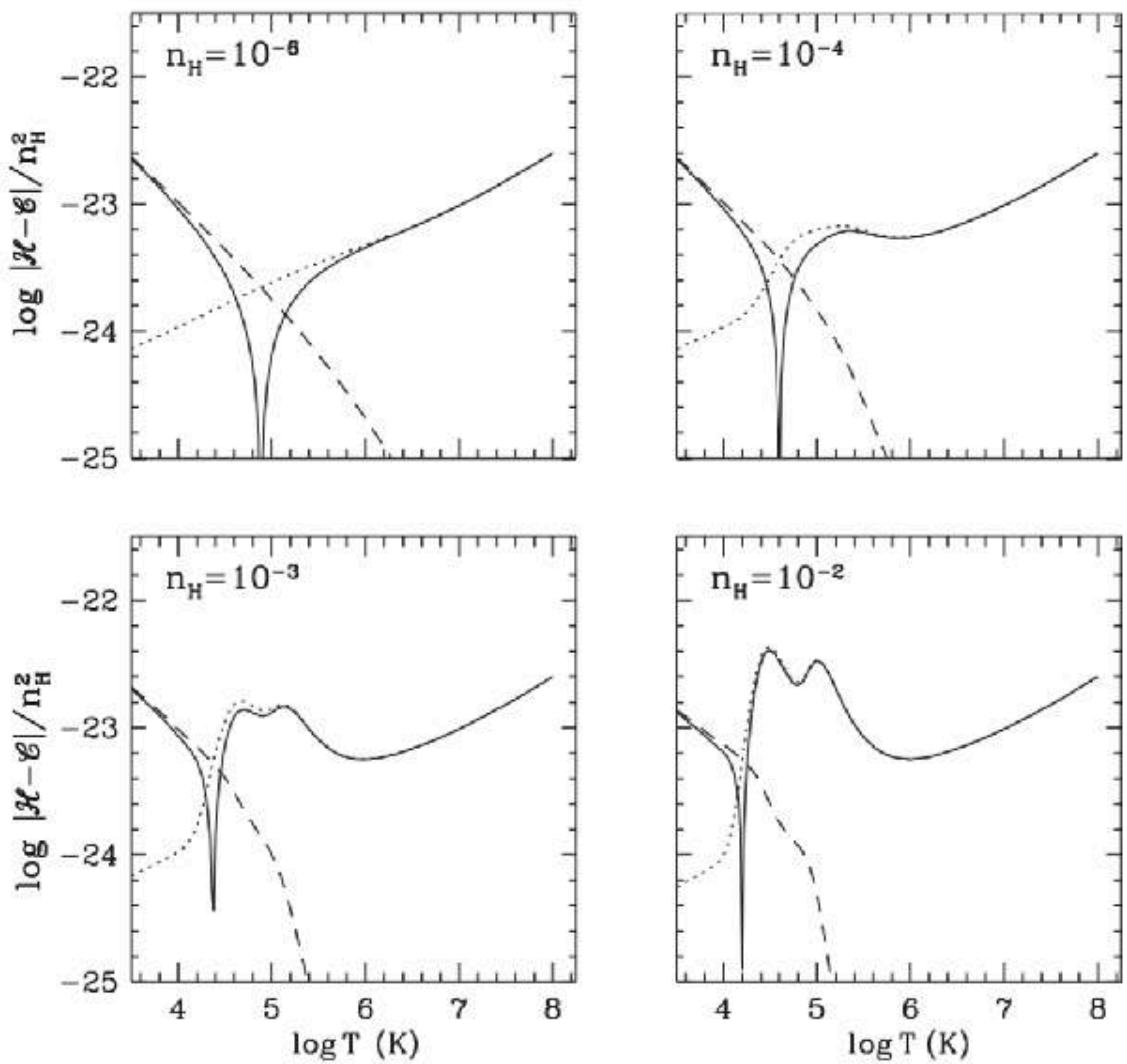
# UV background (UVB) radiation

specific intensity: $$J_{\nu_0}(z_0) = \frac{c}{4\pi}\int_{z_0}^{\infty} \mathrm{d}z \, \frac{(1+z_0)^3\,\epsilon_\nu(z)}{(1+z)\,\mathrm{H}(z)}\, \mathrm{e}^{-\tau_{\rm eff}(\nu_0,\, z_0,\, z)}.$$




Haardt & Madau ’96, ’11; Faucher-Giguere+’09; Khaire & Srianand ’19; …

# Net cooling rate with heating




solid line:
net rate
[erg cm$^3$ s$^{-1}$]

dotted:
cooling rate

dashed:
photo-ioniz.
heating rate

Weinberg+'97

With UVB: $J(\nu) = 10^{-22}(\nu_H/\nu)\,\mathrm{erg\,s^{-1}\,cm^{-2}sr^{-1}Hz^{-1}}$.

# Useful packages for cooling

**Grackle :** https://grackle.readthedocs.io/en/grackle-3.1.1/

(taken out of Enzo AMR simulation)

by B. Smith

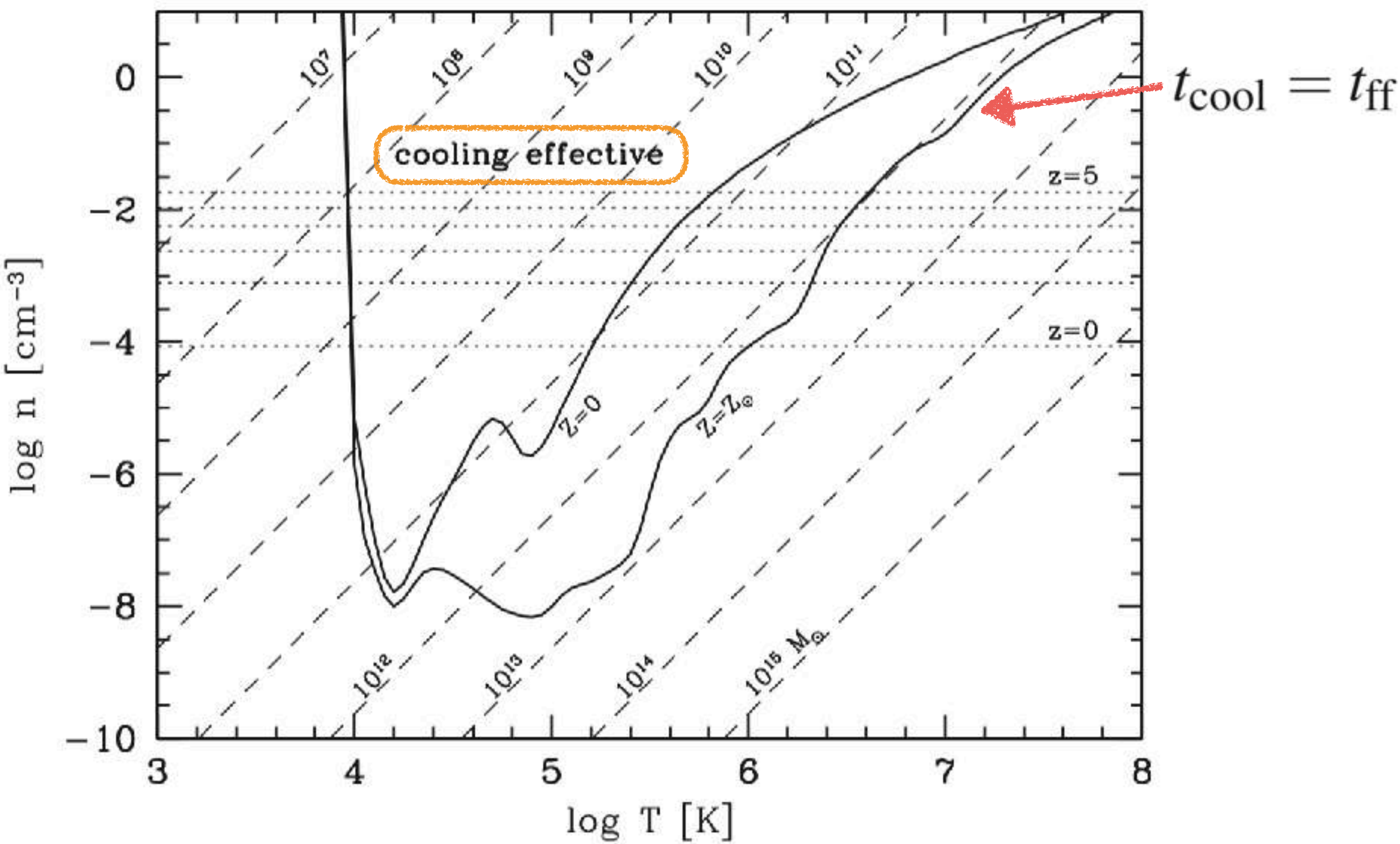
**KROME:** http://www.kromepackage.org/

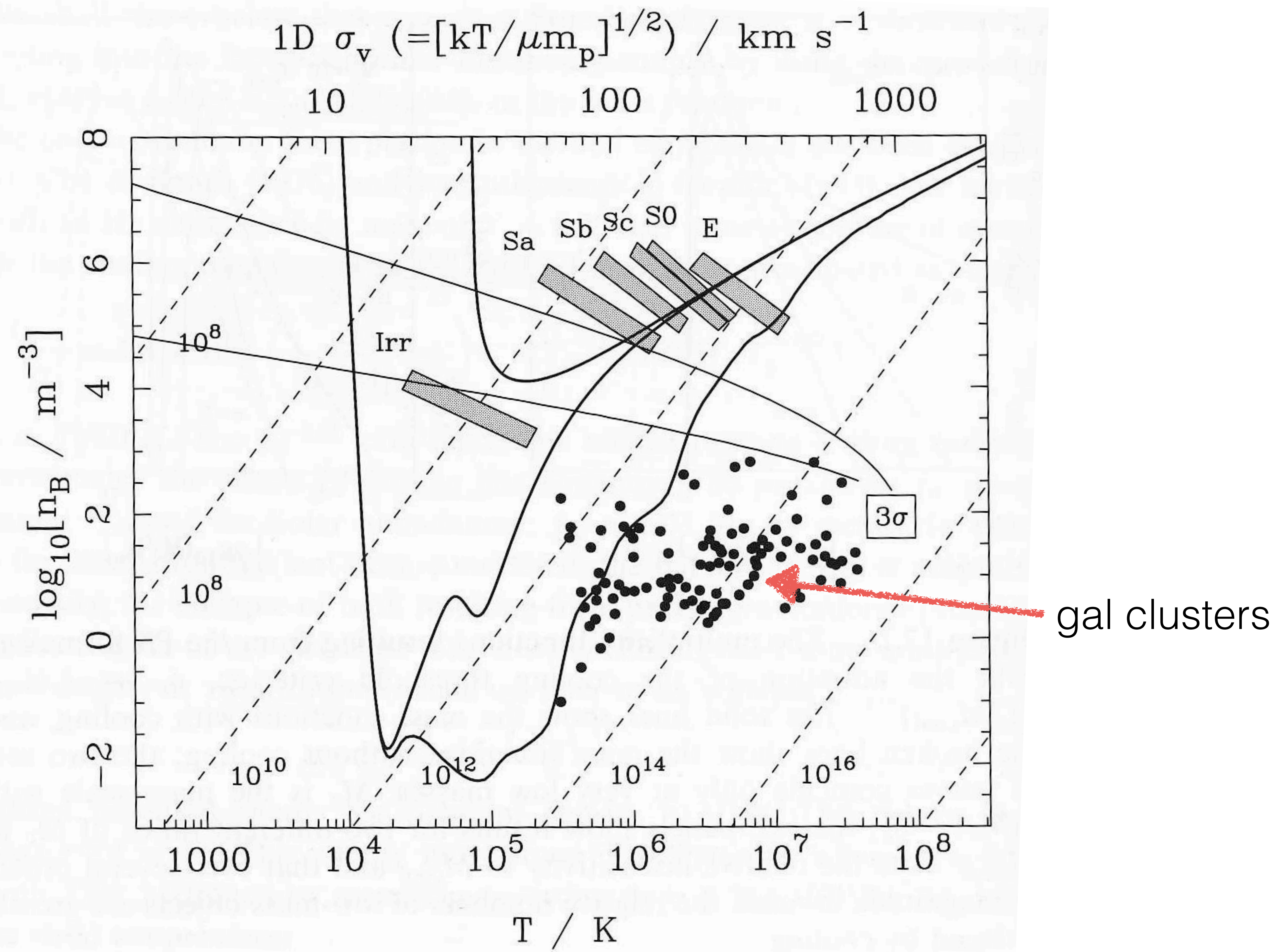
by Grassi, Bovino+

**Cloudy:** https://www.nublado.org/

by G. Ferland+

equilibrium curve






Blumenthal+'84; Peacock textbook, p.572

# Part III.2:
# Star  Formation & Feedback

# “Recipe” for Galaxy Formation

- [x] Background Cosmology
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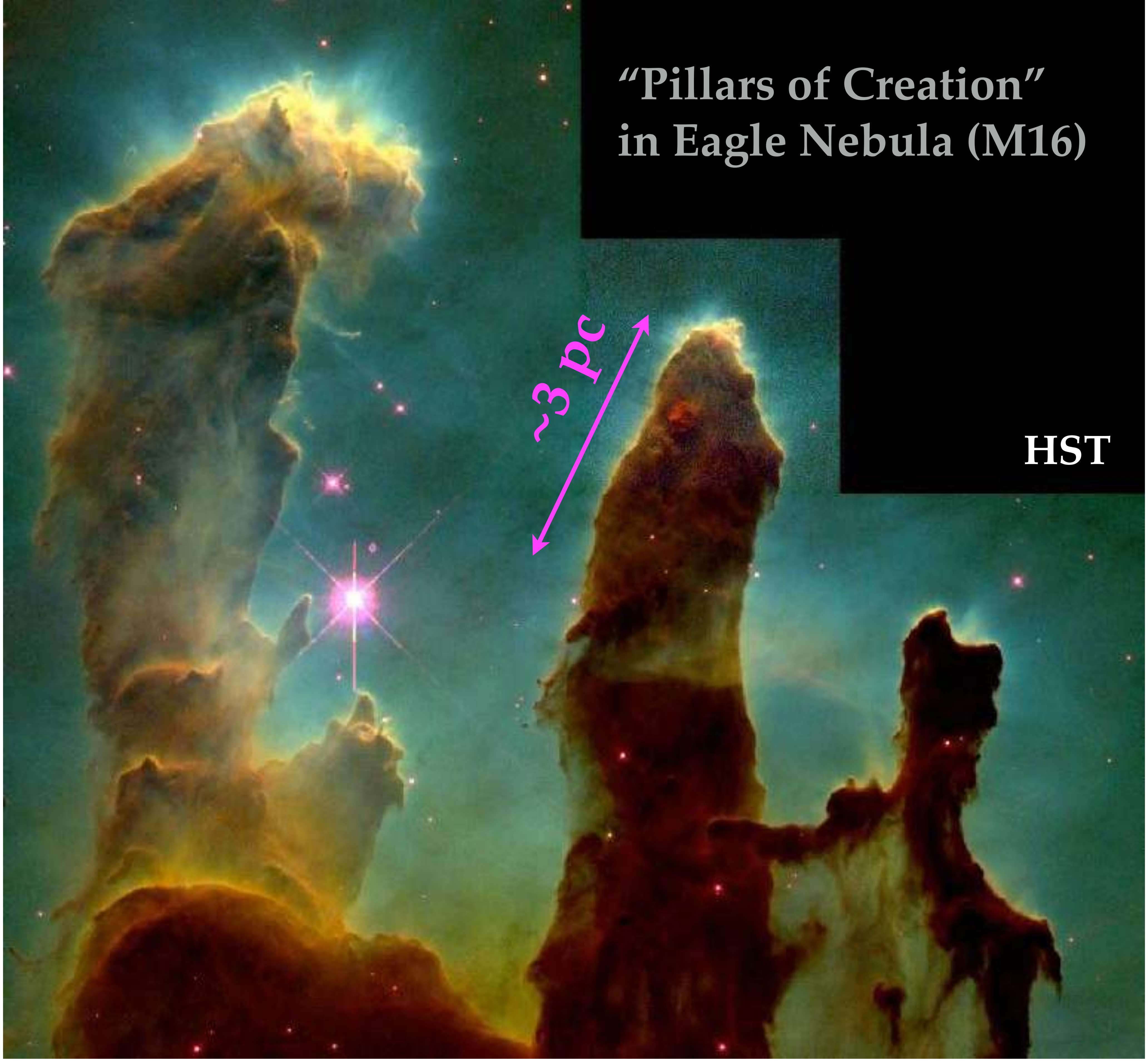
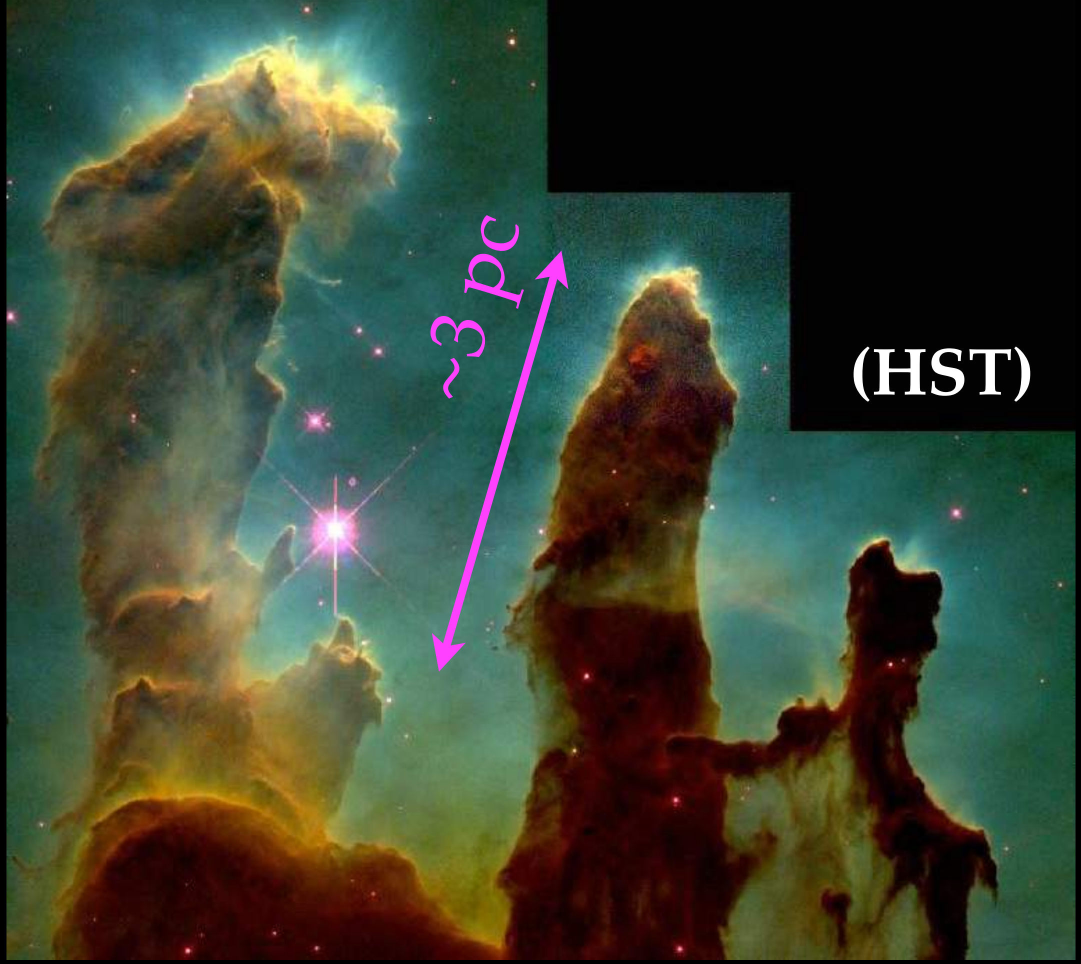
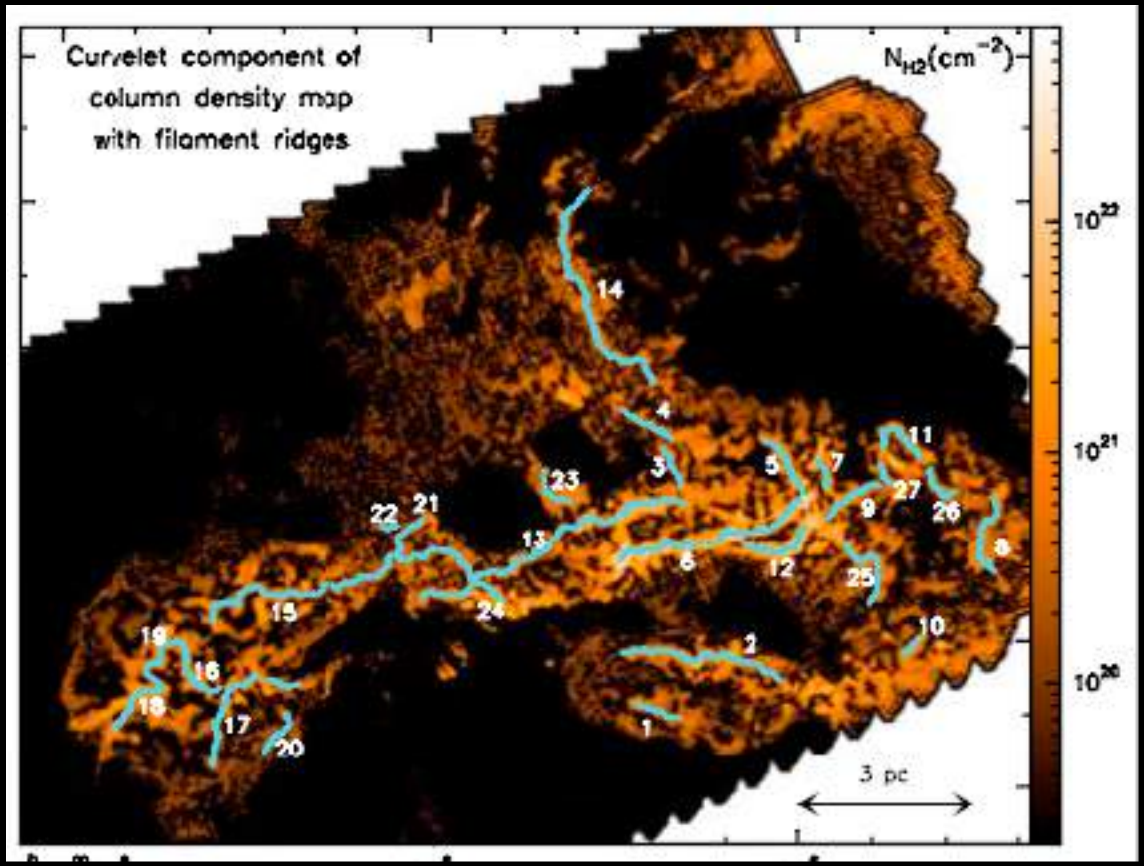

"Pillars of Creation"
in Eagle Nebula (M16)
~3 pc
HST




`Pillars of Creation' in Eagle Nebula (M16)

- Current cosmological simulations lack the ***spatial*** and ***mass*** resolutions to resolve the small scale processes which govern star formation (SF) within the ISM.

- **Need a subgrid model for SF**




- **IC5146 molecular cloud**

**Filament thickness: ~0.1 pc**
**($N_H \gtrsim 10^{22}$ cm$^{-2}$)**

**(~sonic scale below which interstellar turbulence becomes subsonic in diffuse gas)**

**Herschel 70-500$\mu$m (Arzoumanian+11)**

# Star Formation model

$$\boxed{\begin{aligned} \delta &> \delta_{\rm SF} \quad \text{(overdense)} \\ \nabla \cdot \boldsymbol{v} &< 0 \quad \text{(converging gas flow)} \\ t_{\rm cool} &< t_{\rm dyn} \quad \text{(cooling fast)} \\ m_b &> m_J \quad \text{(Jeans unstable)} \end{aligned}}$$

Katz & Gunn '92

Cen & Ostriker '93

then, spawn star ptcl according to $m_\star = m_b \Delta t / t_{\rm dyn},$

For SPH: $\dfrac{h_i}{c_i} < \dfrac{1}{\sqrt{4\pi G \rho_i}},$ $\Rightarrow$ $\boxed{\dot{\rho}_\star = \epsilon_\star \dfrac{\rho_{\rm gas}}{t_{\rm dyn}},}$

sound crossing time < dynamical time

$$\boxed{\dot{\rho}_\star = (1 - \beta)\, \frac{\rho_c}{t_\star},}$$

Yepes+97; Springel+'03

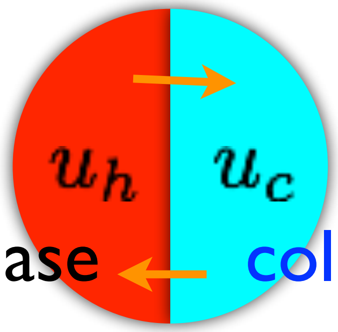
recycling fraction

$\beta \approx 0.1$ (Salpeter) 0.2 (Chabrier IMF)

# Sub-grid Multiphase ISM model

Each SPH ptcl is pictured as a multiphase hybrid gas.

Yepes+97; Springel+'03




hot phase cold phase

$$\rho_h \frac{du_h}{dt} = \beta \frac{\rho_c}{t_\star}(u_{sn} + u_c - u_h) - A\beta \frac{\rho_c}{t_\star}(u_h - u_c) - f\Lambda_{net}$$

(hot phase)

$$u_c = \text{const.}$$

(cold phase)

SFR: $$\dot{\rho}_\star = (1-\beta)\,\frac{\rho_c}{t_\star}$$

cold gas

gas recycling fraction

$$t_\star = t_\star^0 \left(\frac{\rho_g}{\rho_{\rm th}}\right)^{-0.5}$$

$$t_\star^0 = 2.1\,\mathrm{Gyr}$$

$$\boxed{\dot{\rho}_\star \propto \rho^{1.5}}$$

Schmidt law

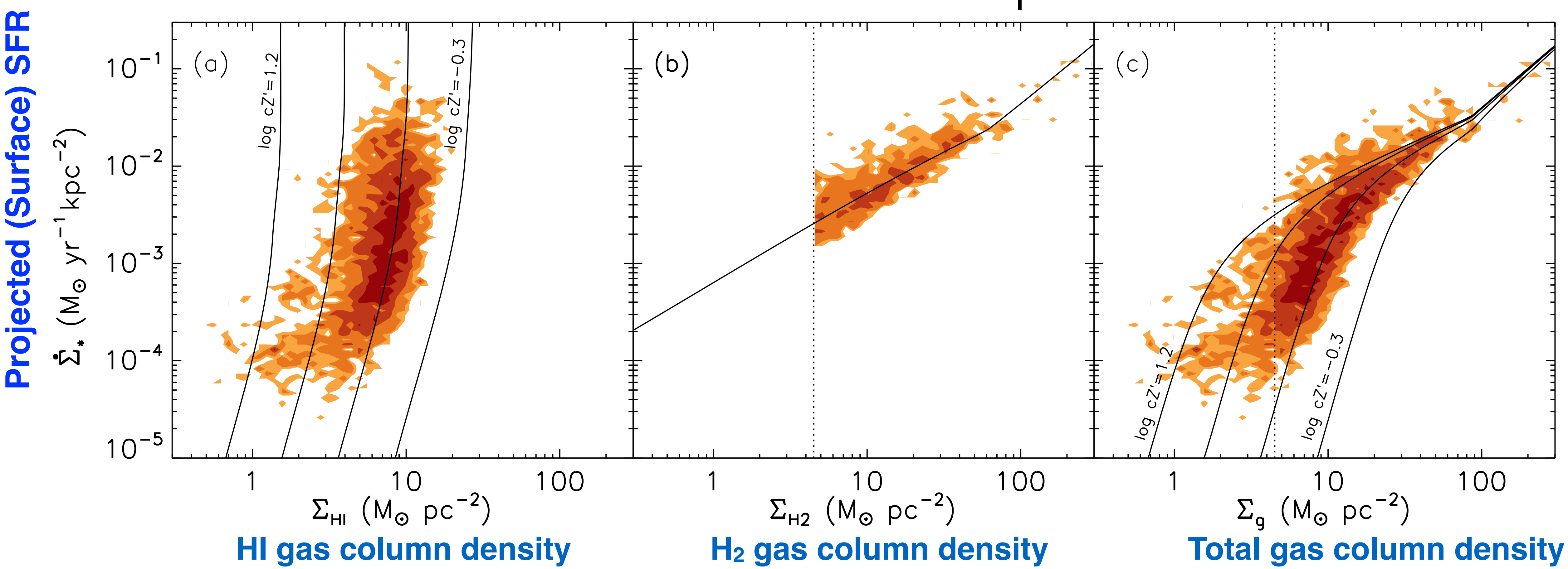
($n_{th} \sim 0.1$ - $1\ \mathrm{cm}^{-3}$)

(controls the normalization; i.e. SF efficiency.)

**But, no apparent dependence on metallicity.**

# $H_2$ dependence of SF

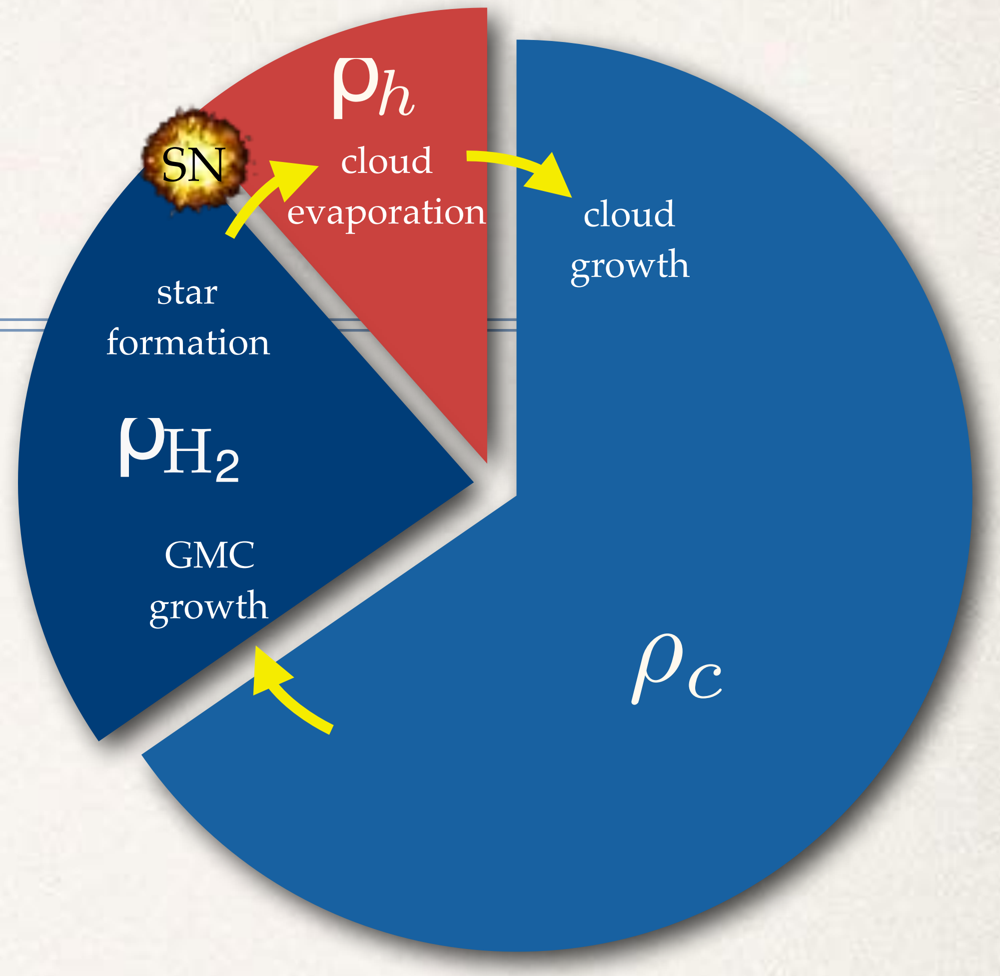
on Kennicutt-Schmit plot




- **SF tightly correlates with molecular gas** (e.g. Bigiel+ '08)
- **Spread can be understood as metallicity dependence** (Krumholz+ '09)

# $H_2$-based SF in SPH

- Modify the multiphase ISM model to include the $H_2$ mass fraction.
- $t_*$ --> ***free-fall time*** of the region.
- SF efficiency: $\epsilon_{ff} = 0.01$
  (Krumholz & Tan '07; Lada+ '10)




$$\dot{\rho}_* = (1-\beta)\epsilon_{ff}\frac{\rho_{H_2}}{t_*}$$

where $$t_\star = t_{ff} = \sqrt{\frac{3\pi}{32G\rho_{gas}}}$$

β: Instantaneous Recycling Fraction

(β≈0.2 for Chabrier IMF)

Thompson, KN+ '14

(cf. Christensen+; Gnedin+, Robertson+,…..)

# "Recipe" for Galaxy Formation

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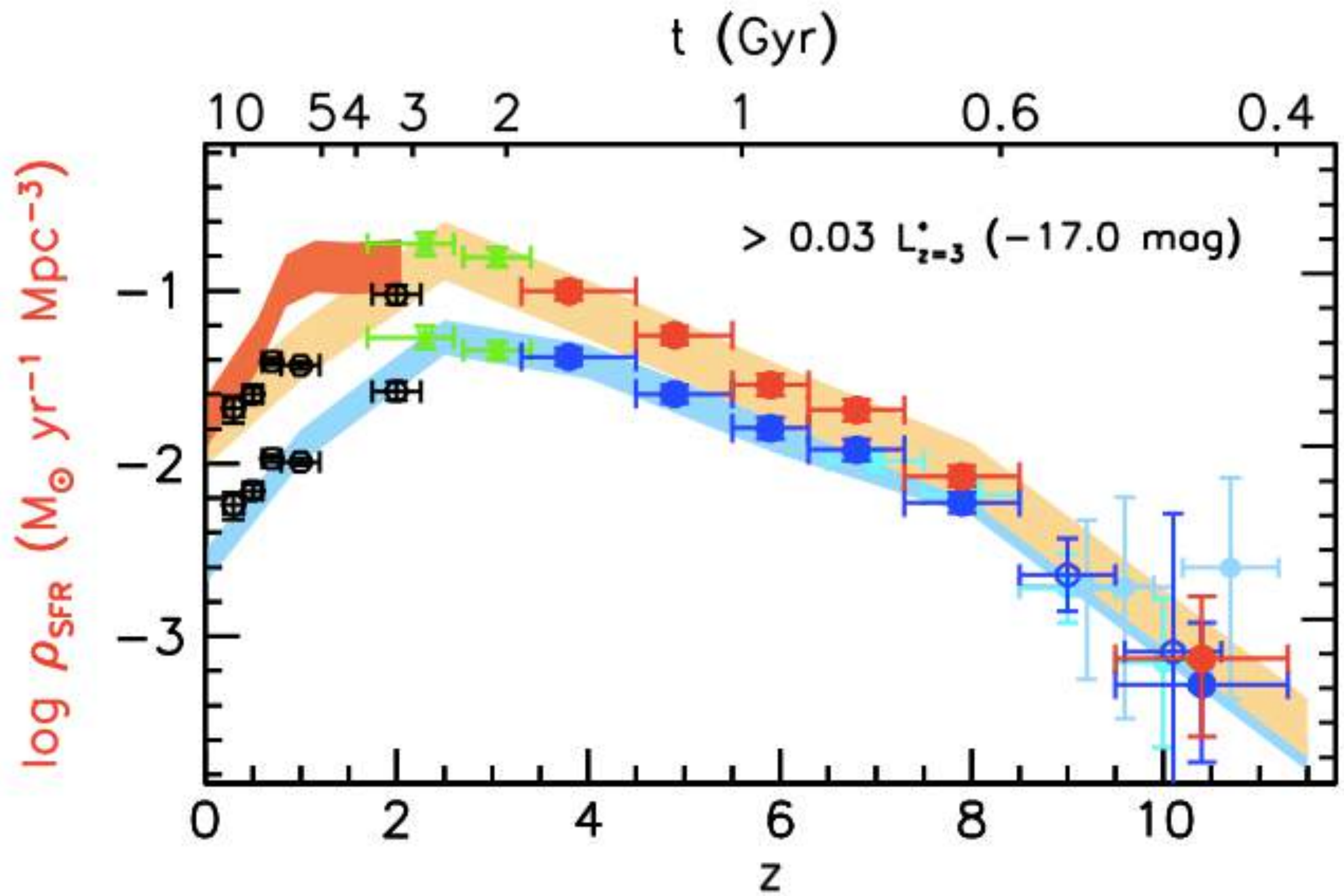
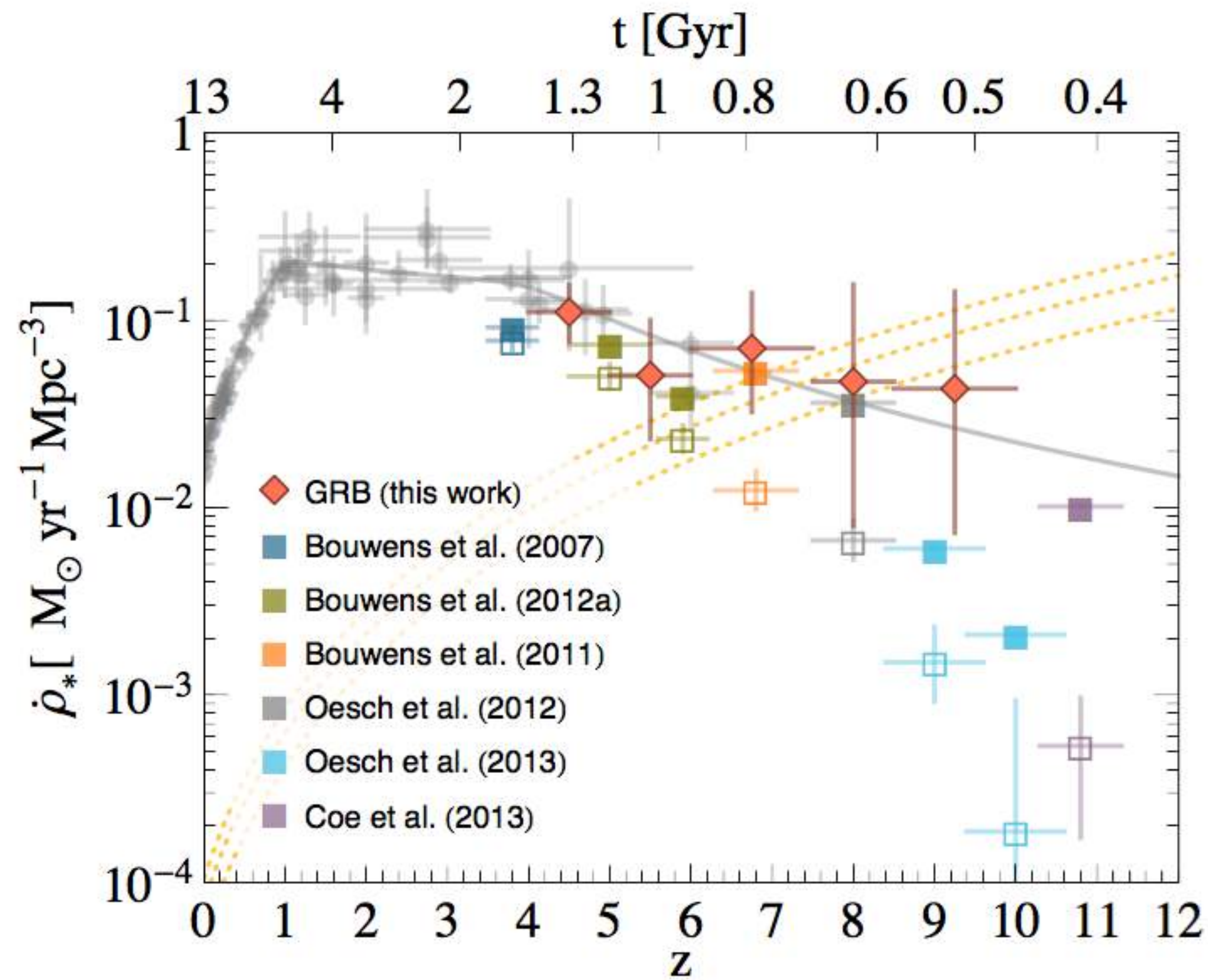
- ☑ Star Formation
- ☐ Feedback (SNe, AGNs), Chemical Enrichment

# Cosmic Star Formation History

## — Observation —




Bouwens+'09, '15

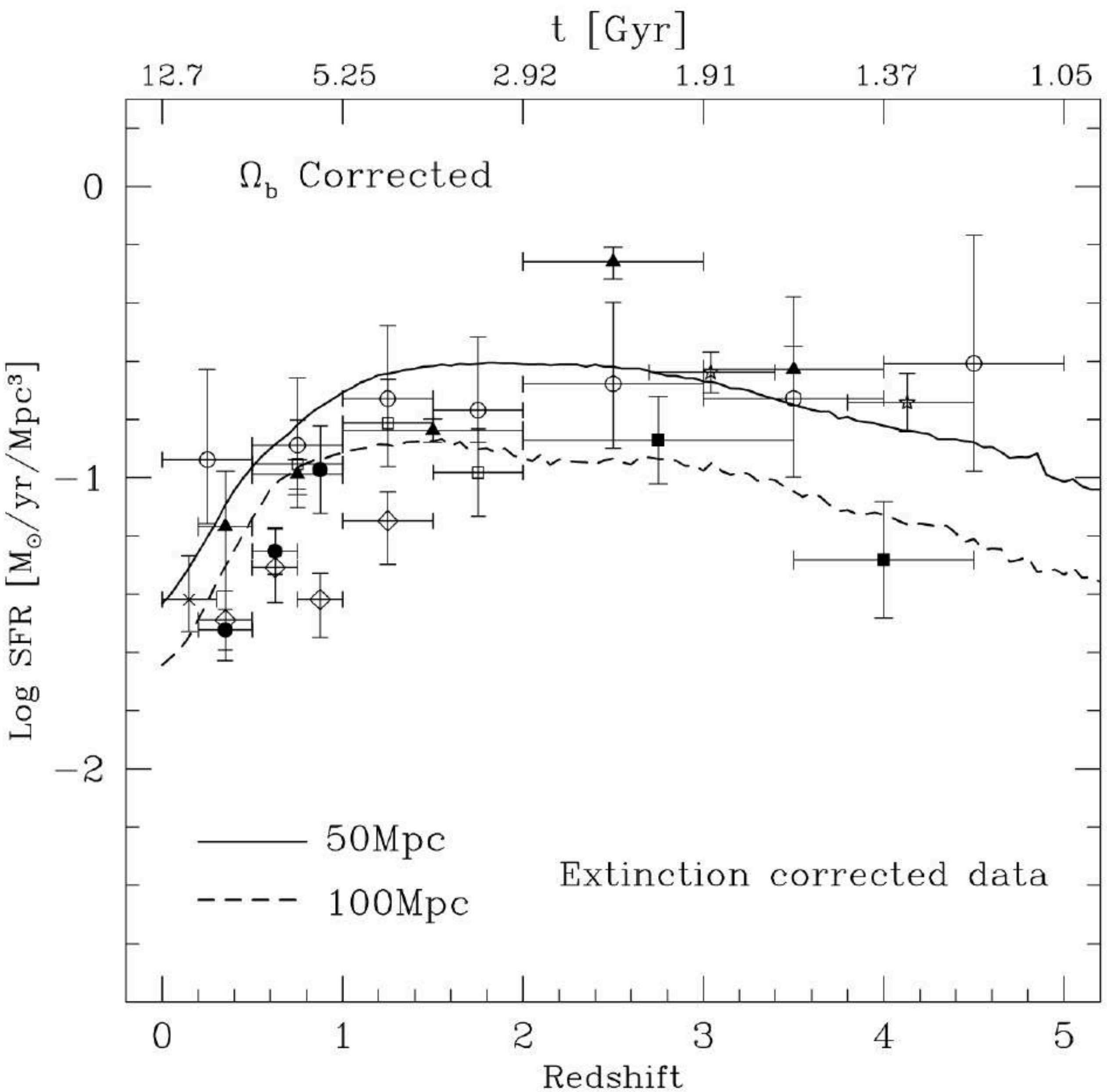



Kistler+'13

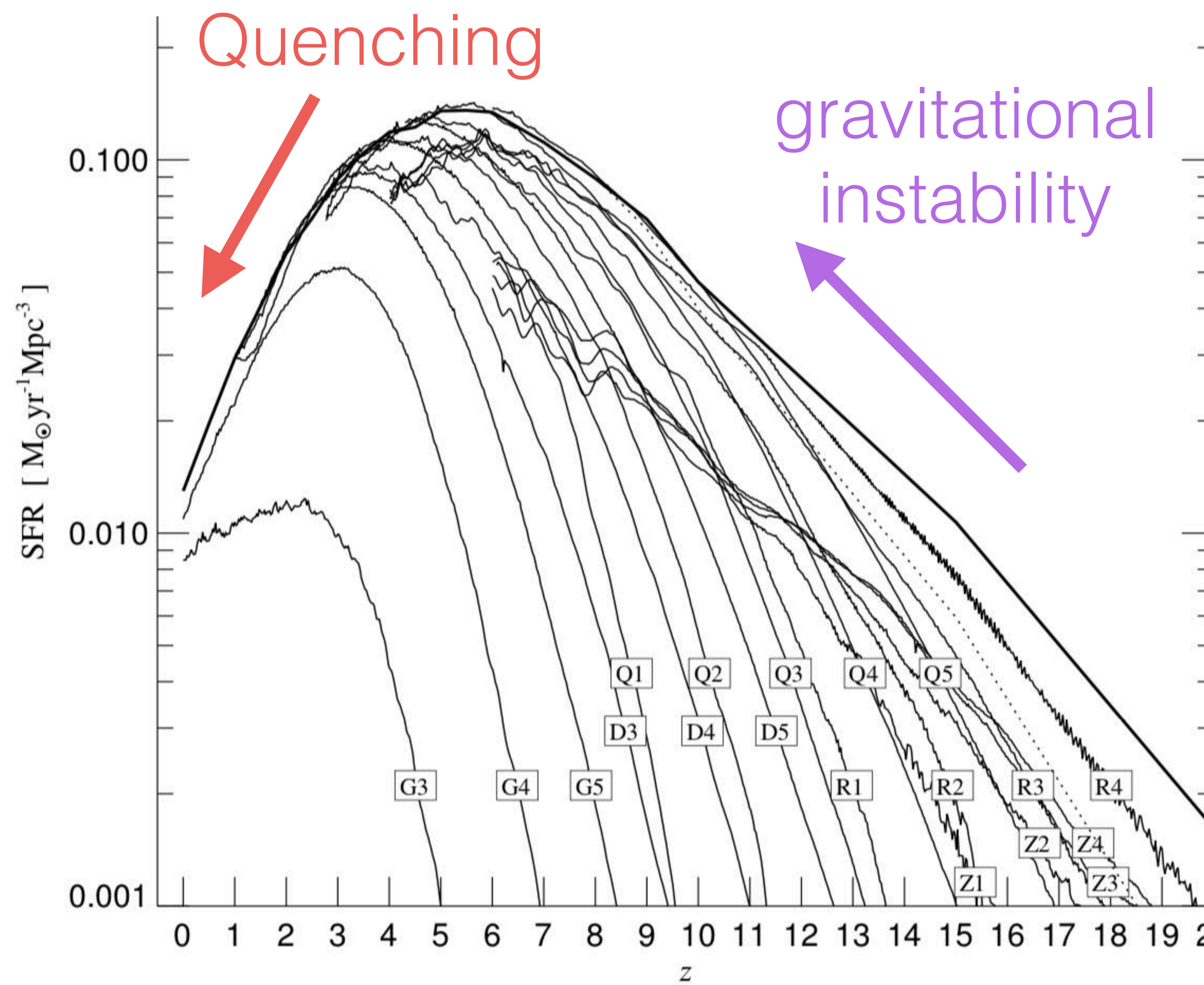
**How many stars are being formed per yr per unit volume.**

# Cosmic SFRD

## — Simulation —




KN, Cen, Ostriker ‘00



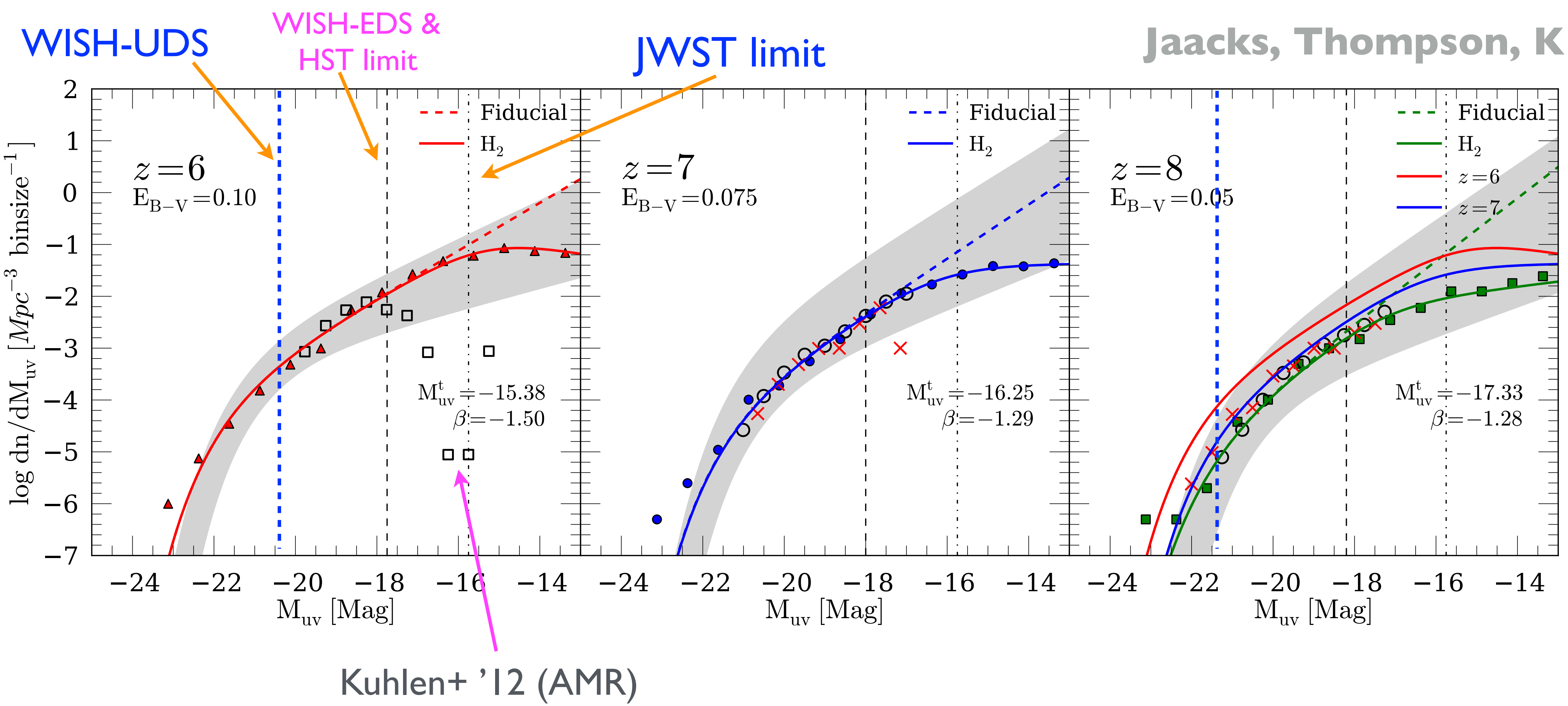

Springel & Hernquist '03

Quenching due to: **- cosmic expansion**

**- feedback**

# High-z LFs with $H_2$-SF model

Jaacks, Thompson, KN '13




(cf. O'Shea, KN+'05: Enzo-Gadget comparison)

Modified Schechter Func.

$$\Phi(L) = \phi^* \left(\frac{L}{L^*}\right)^{\alpha} \exp\left(-\frac{L}{L^*}\right) \left[1 + \left(\frac{L}{L^t}\right)^{\beta}\right]^{-1},$$

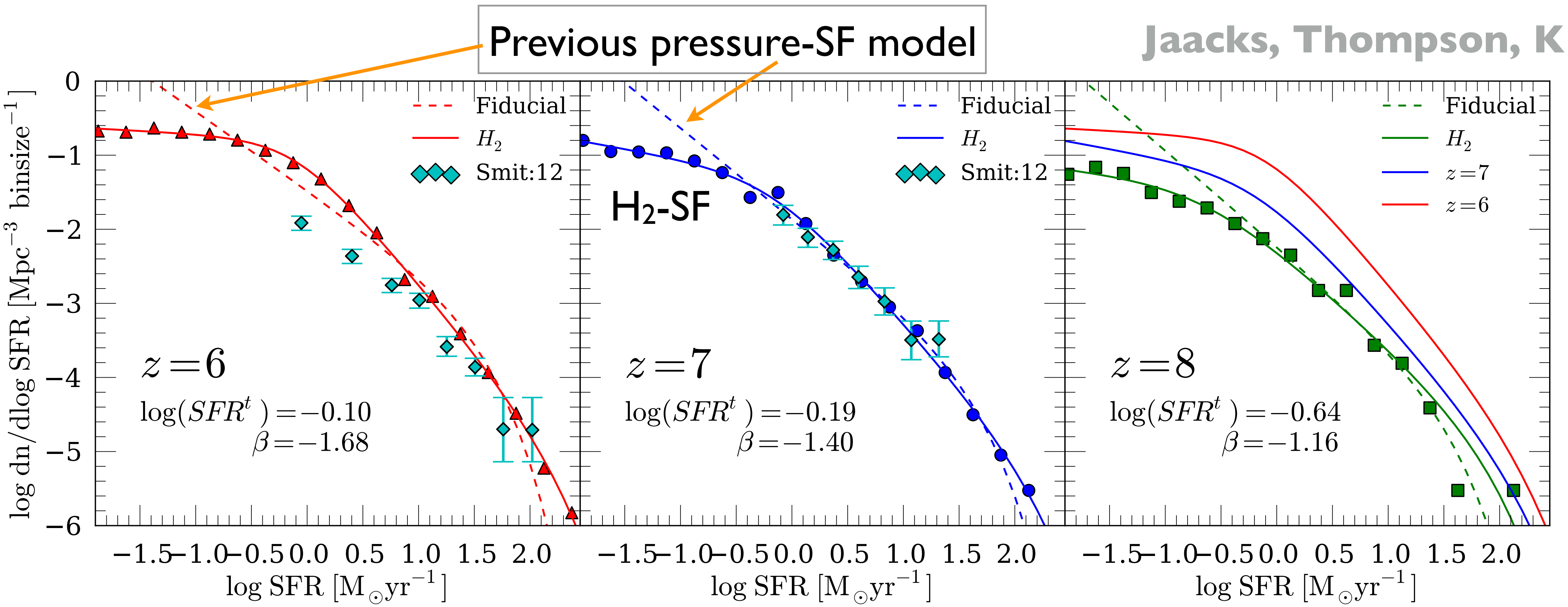
(cf. Loveday+ '97)

**# of low-mass gals is significantly reduced at $M_{uv}$>-16**

**Future test with JWST.**

# SFR fcn w/ $H_2$-SF model

Jaacks, Thompson, KN '13




Modified Schechter SFR fcn:

$$\phi(\mathrm{SFR}) = \ln(10)\phi^* \left(\frac{\mathrm{SFR}}{\mathrm{SFR}^*}\right)^{(1+\alpha)} \exp\left(-\frac{\mathrm{SFR}}{\mathrm{SFR}^*}\right) \times \left[1 + \left(\frac{\mathrm{SFR}}{\mathrm{SFR}^t}\right)^{\beta}\right]^{-1},$$

**Agrees well with current obs constraints at z=6 & 7 (Smit+ '12)**

**SFR fcn provides more direct comparison btw sim & obs.**